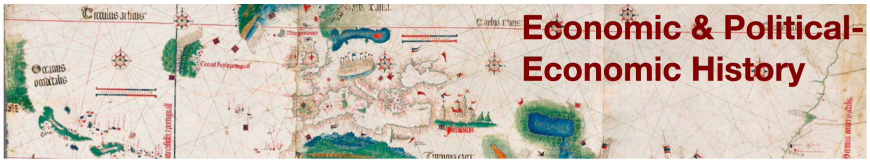

The History of Economic Growth

The Solow-Malthus Model

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2.2. The Solow-Malthus Model

Two major changes to the Solow model are needed in order to make it useful for making sense of the pre-industrial past. The first is to make labor efficiency depend on the scarcity of resources. The second is to make the rate of population and labor force growth depend on the economy's prosperity. We call the changed model that results from these changes the "Solow-Malthus" model.

2.2.1. Review

Recall we started with our Solow growth model:

$$Y = \kappa^{\theta} E L$$

Total production Y is capital-intensity κ as measured by K/Y, the quotient of the capital stock K with production, raised to the salience-of-capital-in-production parameter θ times the labor force L times the efficiency of labor E.

In this Solow model, the equilibrium capital-intensity of the economy is:

$$\kappa = \frac{s}{n + g + \delta}$$

where n is proportional rate of growth of the labor force L, g is the proportional rate of growth of labor efficiency E, s is the gross savings and investment share of production, and δ is the depreciation rate on capital. (Why is the default baseline case one in which the gross savings rate is to no degree linked to the depreciation rate, as if people don't recognize the existence of depreciation at all when they save and invest? Thomas Piketty is perhaps the loudest at (rightly) complaining about this feature of our models. I asked Bob

Solow once why he had done this in his papers back in the 1950s: he shrugged, and said "referees".)

The efficiency of labor E has growth rate g because of improving technology levels H themselves growing at rate h.

And production-per-worker y is simply $y=Y/L$.

Note that is probably best to take a very expensive definition of the capital stock K and capital intensity κ here. It is not just machines and structures. It is everything that is not (a) technological knowledge—ideas about how to manipulate nature and organize humans at a small scale for cooperation—as impeded by (b) resource scarcity. Thus much of what is called "Smithian growth"—an extended and intensified division of labor arising from density of settlement and activity and ease of commercial exchange that allows for higher productivity falls under an increase in κ: it is not just the physical capital of the firm, it is infrastructure capital of the public sector and the social trust and law-and-order capital of the society, and investments in those are as much part of I as is the construction of a factory.

2.2.2. Solow-Malthus Basics

2.2.2.1. Population, Resource Scarcity, and the Efficiency of Labor

Back before the Industrial Revolution and Modern Economic Growth, however, labor efficiency E and thus average output-per-worker $y = Y/L$ were typically stagnant: $g=0$. Yet technology—useful ideas about how to manipulate nature and organize humans—H was not stagnant, but growing at some rate h.

How can we model this? We start to model this by, first, making the efficiency of labor E a function not just of the level of technology H but also of available natural resources per worker R/L. We do this by setting the rate of efficiency of labor growth g equal to the difference between the rate h at which economically useful ideas are generated, and the rate of population and labor force growth n divided by an effect-of-resource scarcity parameter γ, because a higher population makes natural resources per capita increasingly scarce. Therefore:

$$\frac{1}{E}\frac{dE}{dt} = \frac{d\ln(E)}{dt} = g = h - \frac{n}{\gamma}$$

Thus:

$$\frac{dy^{*mal}}{dt} = 0 \text{ ; whenever } h - \frac{n}{\gamma} = 0$$

$$n^{*mal} = \gamma h$$

is the population growth rate at which:

$$\frac{dy^*}{dt} = 0$$

When population is growing at the rate n^{*mal}, the efficiency of labor—and thus the steady-state growth-path level of production per worker Y/L—is constant. This captures the idea that even though human technology was advancing over the ten millennia before the Industrial Revolution, living standards were not because the potential benefits from technology and organization for productivity were offset by the productivity-diminishing effects of smaller farm sizes and more costly other natural resources to feed and provide for the growing population.

2.2.2.2. Determinants of Population and Labor Force Growth

We also need to make the rate of growth of the population and labor force depend on the level of prosperity $y = Y/L$; on the "subsistence" standard of living for necessities y^{sub}; and also on the fraction $1/\phi$ of production that is devoted to necessities, not conveniences and luxuries, and thus enters into reproductive and survival fitness. The higher the resources devoted to fueling reproductive and survival fitness, the faster will be the rate of population growth:

$$\frac{1}{L}\frac{dL}{dt} = \frac{d\ \ln(L)}{dt} = n = \beta\left(\frac{y}{\phi y^{sub}} - 1\right)$$

Then for population to be growing at its Malthusian rate:

$$\gamma h = \beta\left(\frac{1}{\phi}\right)\left(\frac{y}{y^{sub}} - \phi\right)$$

Thus there is an equilibrium "Malthusian" level of production per worker:

$$y^{*mal} = \phi y^{sub}\left(1 + \frac{n^{*mal}}{\beta}\right) = \phi y^{sub}\left(1 + \frac{\gamma h}{\beta}\right)$$

2.2.2.3. The Meanings of the Parameters

Note that these demographic equations only hold for poor populations—one that have not gone through the demographic transition. When populations grow rich and literate enough—and when women acquire enough social power—human societies undergo the demographic transition: women limit their pregnancies to the number of children they desire, confident that they will pretty much all survive to outlive them. Beyond a certain income level, these equations no longer holds. But they did hold up until well after the start of the Industrial Revolution.

Note also the expansive definition of the "conveniences and luxuries" parameter ϕ. It is everything that drives a wedge between the production of society and the typical level of consumption of things that assist biological reproductive fitness in a Darwinian sense, a well-made and well-decorated pot that does not allow you to get more nutrients into your body so you can successfully reproduce is in ϕ. So is a rapacious upper class that hogs resources. So is a taste for living in cities: places where mortality is high because sanitation is even worse than usual, diseases cann more easily jump from person to person, and so plagues can burn hotter and the endemic disease load can be higher.

And, most important of all, note the very crucial role of y^{sub}, the "subsistence" level of average necessities consumption. If the *mores* of your society delay female first marriage, that raises y^{sub}. If whatever local army exists cannot keep barbarians from pillaging and killing, that raises y^{sub}. If your society engages in large scale female infanticide, or simply does not feed girls until after the boys have been fed, that raises y^{sub} (boys, after all, cannot grow up to bear children). If any substantial chunk of your female population remain celibate for their lifetimes, that raises y^{sub}.

It is very important to register this: The Malthusian equilibrium level of production per worker is in no way a constant or even a "natural" phenomenon. It is *sociological*, or socio-political, more than bio-economic. Yes, γ—the effect of resource scarcity on production—and β—how much at the margin extra necessities consumption boosts the population growth rate—can be thought of as, mostly, given by nature's technological and biological constraints. But more important: y^{sub} and $1/\phi$ (and also the rate of invention and innovation h) are overwhelmingly *sociological*: not "natural" at all. And definitely not constant across societies, or over ages.

2.2.3. The Full Malthusian Equilibrium

Then with these added to our Solow growth model to turn it into the Solow-Malthus model, we can calculate the full Malthusian equilibrium for a pre-industrial economy. We can determine the log-level *ln(E)* of the efficiency of labor:

$$\ln(E) = \ln(H) - \frac{\ln(L)}{\gamma}$$

Then since:

$$y^{*mal} = \left(\frac{s}{\gamma h + \delta}\right)^{\theta} E$$

From:

$$\ln(\phi) + \ln\left(y^{sub}\right) + \ln\left(1 + \frac{\gamma h}{\beta}\right) = \theta \ln(s) - \theta \ln(\gamma h + \delta) + \ln(E)$$

The (ln) population and labor force in the full Malthusian equilibrium will be:

$$\ln(L_t^{*mal}) = \gamma\left[\ln(H_t) - \ln(y^{sub})\right] + \gamma\theta\left(\ln(s) - \ln(\delta)\right) - \gamma\ln(\phi) + \left(-\gamma\theta\ln(1 + \gamma h/\delta) - \gamma ln\left(1 + \frac{\gamma h}{\beta}\right)\right)$$

2.2.4. Understanding the Malthusian Equilibrium

Thus to analyze the pre-industrial Malthusian economy, at least in its equilibrium configuration:

- Start with the rate h at which new economically-useful ideas are being generated and with the responsiveness β of population growth to increased prosperity.
- From those derive the Malthusian rate of population growth: $n^{*mal} = \gamma h$
- Then the Malthusian standard of living is: $y^{*mal} = \phi y^{sub}(1+\gamma h/\beta)$
- And the Malthusian population is:

$$L_t^{*mal} = \left[\left(\frac{H_t}{y^{sub}} \right) \left(\frac{s}{\delta} \right)^{\theta} \left(\frac{1}{\phi} \right) \left[\frac{1}{(1+\gamma h/\delta)^{\theta}} \frac{1}{(1+\gamma h/\beta)} \right] \right]^{\gamma}$$

Thus at any date t, the Malthusian-equilibrium population is:

1. the current level H_t of the valuable ideas stock divided by the (sociologically determined, by, for example western-European delayed female marriage patterns, or lineage-family control of reproduction by clan heads) Malthusian-subsistence income level y^{sub} consistent with a stable population on average, times
2. the ratio between the savings-investment rate s and the depreciation rate δ, raised to the parameter θ which governs how much an increase in the capital-output ratio raises income—with a higher θ, factors like the rule of law, imperial peace, and a cul-

ture of thrift and investment that potentially boost the economy's capital stock will matter more, and can generate “efflorescences"—times
3. the inverse of the conveniences-and-luxuries parameter ϕ—it drives a wedge between prosperity and subsistence as spending is diverted categories that do not affect reproduction, such as middle-class luxuries, upper-class luxuries, but also the "luxury" of having an upper class, and the additional conveniences of living in cities and having trade networks that can spread plagues—times
4. two nuisance terms, both near to one one, which depend on how much the level of population must fall below the true subsistence level at which population growth averages zero to generate the (small) average population growth rate that produces growing resource scarcity that offsets the (small) rate of growth of useful ideas. All this
5. raised to the power γ that describes how much more important ideas are than resources in generating human income and production.

(1) is the level of the stock of *useful ideas* relative to the requirements for subsistence. (2) depends on how the rule of law and the rewards to thrift and entrepreneurship drive savings and investment, and thus the division of labor. (3) depends on how society diverts itself from nutrition and related activities that aim at boosting reproductive fitness and, instead, devotes itself to conveniences and luxuries—including the "luxury" of having an upper class, and all the conveniences of urban life. (4) are constant, and are small. And (5) governs how productive potential is translated into resource scarcity-generating population under Malthusian conditions. And recall the full Malthusian equilibrium standard of living:

$$y^{*mal} = \phi y^{sub} \left(1 + \frac{\gamma h}{\beta}\right)$$

This level of income is:

1. The share of production devoted to luxuries-and-conveniences parameter ϕ, times
2. The level of subsistence parameter y^{sub}, times
3. The (small and constant) nuisance term $1+\gamma h/\beta$ needed to generate average population growth $n^{*mal} = \gamma h$ sufficient for increasing resource scarcity to offset technological progress and so hold productivity and incomes at their Malthusian-equilibrium constant levels.

2.2.5. Implications for Understanding Pre-Industrial Civilizations

Production per worker and thus prosperity are thus primarily determined by (a) true subsistence, (b) the wedge between prosperity and reproductive fitness produced by spending on conveniences and luxuries that do not impact reproductive success, plus a minor contribution by (c) the wedge above subsistence needed to generate population growth consonant with the advance of knowledge and population pressure's generation of resource scarcity.

With this model, we can investigate broader questions about the Malthusian Economy—or at least about the Malthusian model, with respect to its equilibrium:

- How much does the system compromise productivity, both static and dynamic, to generate inequality?
- How would one rise in this world—or avoid losing status relative to your ancestors?
- How does the system react to shocks?:
- like a sudden major plague—like the Antonine plague of 165, the St. Cyprian plague of 249, or the Justinian plague of 542—that suddenly and discontinuously pushes population down sharply…

- like the rise of a civilization that carries with it norms of property and law and commerce, and thus a rise in the savings-investment rate s…
- like the rise of an empire that both creates an imperial peace, and thus a rise in the savings-investment rate s, and that also creates a rise in the taste for luxuries ϕ (and possibly reduces biological subsistence y_{sub} as well…
- like the fall of an empire that destroys imperial peace, and thus a fall in the savings-investment rate s, and in the taste for luxuries ϕ and possibly raises biological subsistence y_{sub} as looting, pillaging, and murdering barbarians stalk the land…
- a shift in the rate of ideas growth…
- a shift in sociology that alters subsistence...

The fall of an empire, for example, would see a sharp decline in the savings-investment share s, as the imperial peace collapsed, a fall in the "luxuries" parameter ϕ, as the taste for urbanization and the ability to maintain gross inequality declined, and possibly a rise in y_{sub}, if barbarian invasions, wars, and social-order breakdown significantly raised mortality from violent death.

This model provides an adequate framework—or I at least, think it is an adequate framework—for thinking about the post-Neolithic Revolution pre-Industrial Revolution economy.

2.2.A. Slides

Things Were Very Different Before 1770 or so…

Since 1770:

- MEG: 1870-2020: h=2.1%/year, g=1.5%/year, n=1.2%/year
- Industrial: 1770-1870: h=0.44%/year, g=0.18%/year, n=0.55%/year

Before 1770: Poverty y* = $900/year (*per capita*) ($1200 Before -6000)

- Rate of ideas growth very slow
- Commercial: 1500-1770: h=0.15%/year, g=0.07%/year, n=0.15%/year
- Late Agrarian: 1-1000: h=0.036%/year, g=0, n=0.072%/year
- Axial Agrarian: -1000 to 1: h=0.061%/year, g=0, n=0.122%/year
- Pre-Civilization Agrarian: -8000 to -1000: h=0.017%/year, g=-0.004%/year, n=0.043%/year
- Gatherer-Hunter: -68000 to -8000: h=0.0027%/year, g=0, n=0.0054%/year

Labor Efficiency E and Ideas H

Labor efficiency E is the result of ideas H and natural resources R per worker L

- Labor efficiency: $E = H(R/L)^{1/\gamma}$
- We will assume γ, the relative salience of ideas vis-a-vis resources, is 2
- A 1% increase in ideas H raises labor efficiency E by 1%
- A 1% increase in resources per worker R/L raises labor efficiency E by ½%
- Labor force growth $n = \frac{1}{L}\frac{dL}{dt}$, ideas $h = \frac{1}{H}\frac{dH}{dt}$, efficiency $g = \frac{1}{E}\frac{dE}{dt}$
- If resources constant, efficiency growth $g = h - n/\gamma$
- Ideas non-rival and non-excludible

Malthusian Models

The salience of capital in determining productivity

The extent to which population depresses productivity

The ratio of knowledge to subsistence income

Nuisance terms

$$L_t^{*mal} = \left[\left(\frac{H_t}{y^{sub}} \right) \left(\frac{s}{\delta} \right)^{\theta} \left(\frac{1}{\phi} \right) \left[\frac{1}{(1+\gamma h/\delta)^{\theta}} \frac{1}{(1+\gamma h/\beta)} \right] \right]^{\gamma}$$

The Malthusian equilibrium population

The ratio of savings to depreciation

The inverse of the taste for luxury

Malthisian equilibrium income level

Sensitivity of productivity to population

True zpg subsistence

Rate of useful ideas creation

$$y^{*mal} = \phi y^{sub} \left(1 + \frac{n^{*mal}}{\beta} \right) = \phi y^{sub} \left(1 + \frac{\gamma h}{\beta} \right)$$

Taste for luxuries

Responsiveness of population growth to prosperity

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