History of Economic Growth

Lecture Notes

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1.1. Introduction: History of Economic Growth

I am, once again, appropriating the intellectual property of the brilliant Melissa Dell, and trying to teach my own version of her course Economics 1342: The History of Economic Growth <https://dell-research-harvard.github.io/teaching/econ1342>.

Economics 1342: The History of Economic Growth

Offered Fall 2019

This course examines the history of economic growth, beginning with the divergence between human ancestors and other primates and continuing through the end of the 20th century. Topics covered include the Neolithic Revolution; economic growth in ancient societies; the origins of modern economic growth; theories and evidence about the institutional, geographic, and cultural determinants of growth; the East Asian Miracle; the middle income trap; the political economy of growth; growth and inequality; and theories and evidence about the persistence of poverty in the world's poorest regions.

Let us begin:

Longest-Run Global Economic Growth

Date	Technological Ideas-Stock Growth Rate h	Technological Ideas Stock Level H (1870 = 1)	Average Annual Real Income per Capita y	Total Human Population L (millions)	Total Real World Income Y (billions)
-48000		0.0256	$1,200	1	$1.20
-8000	0.0011%	0.040	$1,200	2.5	$3.0
-6000	0.011%	0.051	$900	7	$6.3
-3000	0.013%	0.074	$900	15	$14
-1000	0.030%	0.14	$900	50	$45
1	0.061%	0.25	$900	170	$153
800	0.022%	0.30	$900	240	$216
1500	0.052%	0.43	$900	500	$450
1770	0.149%	0.64	$1,100	750	$825
1870	0.442%	1.0	$1,300	1300	$1,690
2010	2.125%	19.6	$10,526	7600	$80,000
2100	2.000%	118.4	$58,518	9000	$526,665

Take the square-root of what we think is the total world labor force ***L*** and multiply it by our very crude measures of average real income per worker ***y***, and call that product "technology"—the value ***H*** of the stock of useful ideas about manipulating nature and organizing humans discovered, invented, developed, and deployed globally into the world economy. Normalize ***H*** by setting its value in 1870 equal to 1.

Why take the square-root? Well, if we just multiplied average income by population—called total world real income "technology" —we would be implicitly assuming that labor is useless and unproductive. But that cannot be right: each mouth comes with two eyes, two hands, and a brain. If we just took average income—called that "technology"—we would be implicitly assuming that natural resources are unimportant, and that it does not matter how small population growth has reduced the size of the average plot of land on which a typical person's food is to be grown. That also cannot be right: resource scarcity, and attempts to compensate for it, are a very real thing in our world, today and in our past.

The square-root is a compromise. Is it the right compromise? We can argue about that. What would you suggest?

There is also—I must admit—worries about the average income estimates. Styles of life and the relative prices of commodities are so different between the upper-middle class of the global north today and Neolithic near-subsistence farmers and Paleolithic gatherer-hunters that a one-dimensional "real income" measure may not have much income. Plus there is the fact that our income measures add things up based on what they cost, but the measure we want is what we spend our income on is worth to us. The difference is what economists call "consumer surplus". (I think I have a memory of economist Robert Barro saying or writing somewhere that for a rival-material good consumer surplus was probably, on average, about equal to factor cost; but that for a non-rival attention-information good the ratio was likely to be much larger: perhaps five or ten to one. That struck me as very smart, and likely to be true. But I have been unable to find this anywhere.)

Nathan Mayer Rothschild, the richest man in the first half of the 1800s, died in his fifties of an infected abscess in his butt—something we would cure with a lancing and a single dose of amoxicillin, followed by being yelled at by a medtech for having let it get so bad before asking for help. Does that mean that every single one of us with access to modern antibiotics is, properly assessed, richer than Nathan Mayer Rothschild was? In a profound sense, yes.

I once got ten pounds of potatoes at Trader Joe's for a dollar. That's 0.005 cents per calorie. At the California minimum wage, that is about 1 second of work for 100 calories. For a Neolithic near-subsistence farmer, about half your work-time has to be devoted just to getting calories, and your productivity is maybe 10000 calories for half-a-day's work: 2500 calories/hour, or 2 minutes for 100 calories. Are we thus not the 12 times that the table above presents but rather 100 times richer on average than our predecessors? Perhaps. But there are also things we value that take just as much of our labor-time to obtain today as 5000 years ago, the respect of our peers high among them. And there are things we value today that our predecessors could not obtain at any price—like amoxicillin. What is the proper average summary statistic of all of these multi-dimensions of economic growth? Is there a proper average summary statistic?

Return to the table with which I began:

Longest-Run Global Economic Growth

Date	Technological Ideas-Stock Growth Rate h	Technological Ideas Stock Level H (1870 = 1)	Average Annual Real Income per Capita y	Total Human Population L (millions)	Total Real World Income Y (billions)
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Focus, for now, on the first two columns—the proportional rate-of-growth, in percent per year, ***h*** of this value-of-technology stock, and its level ***H***.

What jumps out at you from this table? For me, twelve things stand out:

1. **The Neolithic Revolution** from -8000 to -6000: the invention of agriculture (and herding).
2. **The glacial pace of technological progress in the past**—that over 1870 to 2010 we saw, in an average year, 200 times the proportional technological progress of the early Agrarian Age. (And, of course, growth from a much, much higher pace.)
3. Nevertheless, **the large cumulative magnitude of technological progress in the past**: as much from -6000 to 1870 as from 1870-2010.
4. **The acceleration of growth in the early Agrarian Age** -6000 to the year 1: literacy matters for making humanity an anthology intelligence composed not just of those alive today but also of the dead, and two heads are better than one for solving problems.
5. **The Late-Antiquity Pause** from 1 (actually 150) to 800—I at least would have expected another doubling of the rate of progress after the year 1, which would have carried H up to 0.65 by 800 and then possibly put the world at the cusp of the Industrial Revolution, but we did not attain that level of H until 1770.
6. **The Mediæval Recovery** of technological progress to something like its pre-pause norm (but with the year 800 seeing five times the human population of the year -1000, why wasn't there faster growth in H than in the -1st millennium?).
7. **The Imperial-Commercial Age** step-up in growth over 1500 to 1770.
8. **The British Industrial Revolution Age** from 1770 to 1870.
9. **Modern Economic Growth** from 1870 to 2010.

10. **The Population Explosion and Demographic Transition** from 1770 to 2100.
11. **Whatever is going on now**—if global warming and other problems do not interrupt Modern Economic Growth, what do we have to look forward to for the world of 2100?
12. **Is this a misguided intellectual enterprise**—focusing on H, and taking it to be something real and important rather than a distracting mental-fictional cloud-castle that does more to confuse than to enlighten us?

(I figure I can get through this in fifteen minutes at the start, and then see what kind of class the 75 students I will get are—will we then be able to discuss the twelve interesting features of the table (and whatever else they come up with), and for how long, or will I have to start lecturing again?)

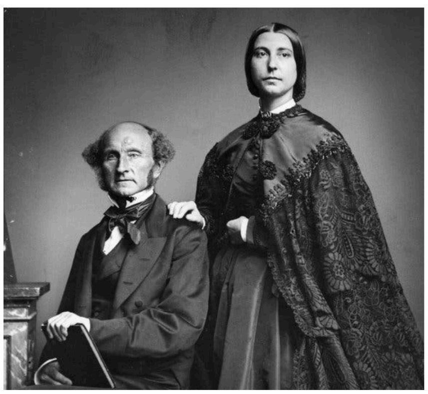
Once again, welcome. I hope you have fun here. I hope you learn stuff. I hope this makes you better citizens, and I hope the knowledge you get here will give you more power over your life and more capability to do useful things for others.

1161 words

1.2. Economic Growth: Idea & Reality

Economic Growth. It is both a reality and an idea. It begins in the mists of deep time, rapidly reaches the invention of agriculture, & continues through with forecasts for the 21st century and beyond.

John Stuart Mill



British polymath:

- “It is questionable if all the mechanical inventions yet made have lightened the day's toil of any human being.
- “They have enabled a greater population to live the same life of drudgery and imprisonment…”

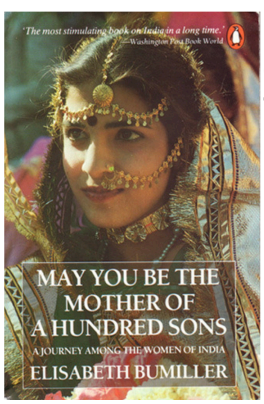
Let me start back in 1870, with British public intellectual, journalist, civil servant, imperial bureaucrat, moral philosopher, and economist John Stuart Mill wrote that up to that moment there had been little economic growth. Living standards and productivity levels had been largely stagnant—even in his lifetime. He had seen what people rightly called an “Industrial Revolution”, with the coming of steampower, automatic machinery, factories, railroads, and telegraphs. And yet, he wrote:

> It is questionable if all the mechanical inventions yet made have lightened the day's toil of any human being. They have enabled a greater population to live the same life of drudgery and imprisonment…

The benefits of invention and innovation had all flowed to the top, and allowed "an increased number of manufacturers and others to make fortunes..." He did, almost as an aside, agree that "they have increased the comforts of the middle classes..."

But, in Mill's eyes, the human economy in 1870 was largely as it had been for the nearly 8000 years since the invention of agriculture: people were desperately poor, with advancing technology barely keeping pace with increased resource scarcity generated from larger populations. People were so malnourished and disease-ridden that for a couple to have on average two children surviving to reproduce required having three children live through to adulthood, which required that 4.5 children survive to the age of 5, which required 8 pregnancies carried to or near full-term. And humanity was at or at least near the demographic limit.

Malthusian Demography



Social power depends on being the mother of sons:

- But the average mother has only one son surviving to reproduce
- Some have two or three, and some have zero
- Hence very strong pressure to have as many as possible, in the hope that one will survive

- Two children survive to reproduce
- Three live to adulthood
- 4.5 survive to age 5
- 6.5 live births
- 9 pregnancies, with miscarriages and stillbirths

9 x 9 = 81/12 = more than 6 years pregnant

15 years breastfeeding

21 years eating for two

And now let us jump back a further 2200 years, to the time of Aristoteles of Stageira, sometime tutor of Alexandros III Argeádai of Macedon, called “The Great”. For 2000 years From the moment he became the favored pupil of Plato up until, I am not sure, Call it the year 1650, and in a long arc from Ireland to India, Aristotle was THE Philosopher. Capital P. THE definite article. If you said “the philosopher”, you were referring to Aristotle. And people did. He was “the master of those who know”, as Florentine poet Dante Alighieri named him.

“THE Philosopher”

For, literally, millennia…

- “the master of those who know”, as Dante called him…
- Interested in *everything*—except economic growth
- Scroll I of his *Politics*
  - “household management” = oiko-nomos = economics
  - In order: bossing slaves, raising children, directing your wife, knowing market conditions

Aristotle was very interested in almost everything – except economic growth. Aristotle’s main discussion of economics comes in the book we call the politics. The *Politics* is about how prosperous and wealthy Greek men organize themselves and their inferiors into city-states that provide an arena and support for life and, of course, for the practice of philosophy. The first book—actually, for him it was the first scroll—Of the politics is about economics, or rather resources and household management, because unless resources and the households Controlled by prosperous and wealthy

Greek men Are present and well organized, successful organization of a city state, of a polity, will be impossible.

In the first book of his *Politic*s, Aristotle talks about the necessity of owning slaves. It is, in fact, The first thing on his mind when he talks about managing resources on the part of the household—Greek oikos, household, and Greek nomos, organization or management. Hence oiko-nomos. Hence economics. THE Philosopher says:

> Let us first speak of master and slave…. No man can live well, or indeed live at all, unless he be provided with necessaries…. Just as must have their own proper instruments... so it is in the management of a household… [which needs] property as instruments for living. And... a slave is living property.… If every tool could accomplish its own work, obeying or anticipating the will of others, like the statues of Daidalos, or the tripods of Hephaistos, which, says the poet Homer, "of their own accord entered the assembly of the Gods;" if, in like manner, the shuttle would weave and the plectrum touch the lyre without a hand to guide them, chief workmen would not want servants, nor masters slaves…

The Tripods [Self-Propelled Catering Carts] of Hephaistos…

The tripods of Hephaistos are self-propelled catering carts, from Homer's *Iliad*:

> Thetis of the silver feet came to the house of Hephaistos,
> imperishable, starry, and shining among the immortals,
> built in bronze for himself by the god of the dragging footsteps.
>
> She found him sweating as he turned here and there to his bellows
> busily, since he was working on twenty tripods
> which were to stand against the wall of his strong-founded dwelling.
> And he had set golden wheels underneath the base of each one
> so that of their own motion they could wheel into the immortal
> gathering, and return to his house: a wonder to look at.
> These were so far finished, but the elaborate ear handles
> were not yet on. He was forging these, and beating the chains out.
>
> As he was at work on this in his craftsmanship and his cunning
> meanwhile the goddess Thetis the silver-footed drew near him…

But, concluded Aristotle, since he did not live in such a Golden Age, in which music could be played and cloth woven without human hands; since Aristotle did not have robot blacksmiths or the self-propelled serving trays that could both keep the food warm and decide when it should be brought into the dining room, things that myth attributed to the lifestyles of the heroic and divine—since he did not have these, Aristotle or any other Greek man who wanted to lead a leisurely enough life to have time to undertake philosophy and play a proper role in the self-governance of the city-state needed to own and know how to boss slaves.

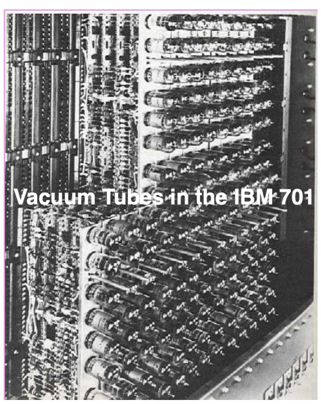
And not just one or two slaves either, but household slaves, agricultural slaves, craft work or slaves, and perhaps more.

For Aristotle, the only way to be wealthy enough to avoid what John Stuart Mill called "the same life of drudgery and imprisonment" was to have captured people and then dominated them to take for yourself the stuff that they produced. The idea that there might be economic growth was simply not on his radar screen. And so for Aristotle, it was a grave mistake that the richest city of his age—Athens—allowed the richest sixth or so of its adult population a role in voting on what the policy of the city should be. In his view, there was no way such a large proportion of the population could have been well-educated enough and possess enough time to have an intelligent view on political questions.

Today is very different.

We look forward to achieving zero population growth in our lifetimes. The average citizen of the world today is 10 times as well-off, at least, as the average citizen of 1870. And we can see the road clear to, in our lifetimes, at least another quadrupling of average human living standards and productivity levels.

Each of These Is a Single Logic Gate

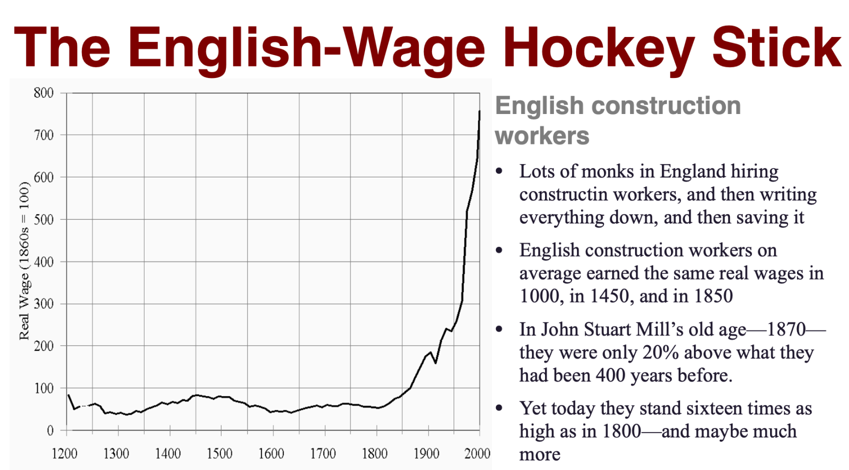


A glass tube filled with a vacuum:

- A NAND gate: 1” in diameter x 4” long
- Today a NAND gate is 100 nm^3
- We could fit 5 x 10^{16} NAND gates inside one of these
- Bottom Line:
  - We produce commodities much more cheaply
  - But we also produce very different commodities
  - Commodities that could not have been produced at any price in 1960 are incredibly cheap today

We have the robot blacksmiths and the robot serving carts. Everywhere in our civilization music is played and cloth is woven without the touch of a human hand. And technology advances still at a furious rate. Here we have a picture of a computer of the 1950s. We today make—or, rather, TSMC in Taiwan and Samsung in South Korea can make—computer electronic switching elements 5 x 10^16, that is, 50 quadrillion times smaller than those made in the 1950s. We produce commodities much more cheaply. But we also produce very different commodities Commodities that could not have been produced at any price in 1960 are incredibly cheap today.

Indeed, if you try to quantify how different our life is from the Life surrounding Aristotle, or even John Stuart Mill, you rapidly find yourself looking at crazy graphs. Like this one. The English-wage hockey stick. English construction workers on average earned the same real wages in 1000, in 1450, and in 1850. In Mill's old age—1870—they were only 20% above what they had been under the Lancastrian dynasty 400 years before. Yet today they stand eight times as high—and maybe much more, as the goods we produce and consume today are so different from those of a century and a half ago that there is a strong sense in which we simply cannot accurately calculate a comparative "real wage".



There has been a huge change. Indeed human life when you reach 60 will be very different from human life when you were born, just as life right now is very different from what life was like when I was born in 1960.

Thus we have some of our big questions: What was life like for the typical person in the year 1870, or -330, or -1130? And why was life for the typical person as late as the year 1870 so similar to life for the typical person in the year -330, or -1130? To what extent do we need to qualify Mill's claim that the working class—even the English working class, the working class in the most technologically advanced and powerful nation the world had ever seen—still "live[d] the same life of drudgery and imprisonment" as had 3000 years ago "another man's ***thes***, a portionless man whose livelihood was small", in the words Akhilleus uses to Odysseus in Homer's *Odyssey* to describe the lowest of lives? And what caused all the big changes from then until now? And what have been their consequences? And how have people viewed this process, and the possibilities for progress or the perceived necessity of stagnation?

Now is a good time for questions. And comments. And questions that really are more of a comment than a question…

1610 words

1.3. Humans & Their Economies: The Eagle-Eye View

Since the invention of agriculture about 10,000 years ago—around the year -8000—there have been three broad eras as far as economic growth is concerned. Call them glacial-frozen, ice-breaking, and high-pressure

The first, glacial-frozen, broad era is that of the Agrarian Age, from the invention of agriculture some 10000 years ago until, well, it did not even begin to end anywhere in the world until 1500. It was a world of peasants, craftsman, priests, and warriors. It was a world of slaves, non-slave non-citizens, citizens, aristocrats, and kings. Call it the Agrarian Age.

All the globe was in the Agrarian Age until the year 1500. More than half the globe was in or close to the Agrarian Age until, perhaps, 1930. It is still present in a non-negligible chunk of the globe today: the bottom billion of our 8 billion people alive today have living standards and productivity levels that are not *that* distinguishable from those of our Agrarian-Age ancestors. The most significant differences are that (i) they have access to moderate amounts of modern public health expenditures, which gives them double or triple the life expectancy at birth of our agrarian-age ancestors; and (ii) they have some access to the global telecommunications network.

In this long Agrarian Age from -8000 to sometime between 1500 and 1930, depending where you lived, humanity (except for a relatively narrow slice of aristocrats at the top of the social hierarchy, and those lucky enough to be able to lick up the crumbs that fell from the aristocrats' tables) lived in what we would regard as dire poverty. They were close to the edge of subsistence, in the sense that had the population been even moderately poorer, the population would not have managed to reproduce itself in the next generation: too many women would have missed ovulation from malnutrition, too many malnourished children with compromised immune systems would have been taken out by the common cold, and too little sanitation made it a truly golden age for plague and dysentery.

Consider: a preindustrial pre-artificial birth-control population that is nutritionally unstressed will triple in numbers every 50 years or so. That was the experience of the *conquistadores* and their descendants in Latin America. That was the experience of the English and French settlers coming in behind the waves of plague and genocide that had decimated the indigenous Amerindian population in North America. That was the experience of the Polish, Ukrainian, and Russian settlers on the Pontic-Caspian steppe, after the armies of the gunpowder empires, most notably of Yekaterina II Holstein-Gottorp-Romanov (neé Sophie),Tsarina of All the Russias, drove out the horse nomads and opened the black-earth regions to the plow.

But back in the Agrarian Age it took the human population not 50 but 1500 years to triple. The population in the year 1500 of 500 million was only three times what it had been in the year one. And yet is there anything that parents would work harder for and spend more effort on than trying to ensure that their children would survive to reproduce? Yet they could not do so, at least not to any extent to make the rate of population growth more than glacial. And note that this was not because of a shortage of births: 8 pregnancies is typical for an Agrarian Age woman. Queen Anne Stuart had 17.

The fact that, in the Agrarian Age, human populations took not 50 but 1500 years to triple a measure of how poor, in the sense of being extraordinarily close to a biological population-sustaining limit humanity was back in the Agrarian Age.

In this Agrarian-Age, from the discovery of agriculture on up to and beyond 1500, civilization–if you want to call it that—depended almost entirely on exploitation. It needed an upper class that took from the peasants and craft-workers enough of the stuff they produced to give the elite a better-than-desperately-poor standard of living, and also allow them, most of the time, to maintain control.

Now this does not mean that the standard of living of the upper class back in the Agrarian Age would really impress us. For one thing, life expectancy was short, even for the upper class: 20 to 30 at birth, rather than our 80 or so. Some plague might will get you. And if you were female, childbed might will get you as well. Of British queens, one in seven in the years from 1000 up to 1650 died in childbed. Males escaped childbed mortality, and also by-and-large escaped the extra mortality from nursing sick children with infectious diseases. But if you were male or unlucky and female you faced risks from human violence. 1/3 of English monarchs from William the Conqueror up to 1650 either died in battle, were assassinated, or were murdered after some sham show of judicial process. Probably the risks of violent death at the hands of others were lower for people who were not so eminent. Probably.

For another, lots of things that we think of as nice to have, but not as especially crucial or key or even valuable, were way out of their reach of even the richest during and after th Agrarian age. Suppose you lived in 1610. Suppose you wanted, in your house, to watch a horror movie—strike that, no movies: a horror play—something about witches or devils or such, and wanted to watch it in your house. One person in all the islnd of Great Britain could do that. If you wanted to do that, you had better be named James I Stuart, you had better be King of England and Scotland, and your acting company had better have either Shakespeare's MacBeth or Marlowe's Dr. Faustus in repertory.

And before: If you were Gilgamesh, King of Uruk in the year -3000, What could you do? You could boss people around yes–get the men to build walls and canals and to drill as soldiers, and get the women to serve and service you. You could drink beer—wine and distillation had not yet been invented. You could eat flatbread or porridge. You could eat meat every day—which your subjects could eat only rarely. You could eat honey—which few people could do. And honey was important: do we have any Melissas or Deborahs out there?

You could sit on cushions stuffed with wool.

But, for Gilgamesh, to even get cedar or some other wood for your walls or floors—that required a major military expedition With associated transport logistics.

Back then, in the Agrarian Age, human technological progress was glacially slow. And here there is a puzzle. For progress and sophistication in the arts, in politics, in religion, in social Organization—ancient Athens even heard its equivalent of keeping up with the Kardashians, only the star was named Phryne and not Kim. But not just in the level of technology but in the pace of technological advance, we moderns far outstrip the ancients in a way that we do not In, say, composing poetry.

And even though life back in the Agrarian Age was nasty brutish and short, it was not always the same. There were civilizational efflorescences. They were empires that brought peace and relative prosperity to large chunks of the land. There were also dark ages and barbarian invasions and civilization-shaking catastrophes as well. There was a history of economic growth and decline in the Agrarian Age: it was not all stasis and stagnation.

Then, around 1500, humanity passes over into a new major watershed. The era of breaking ice begins. The age of the commercial revolution from 1500 to 1770, which, parenthetically, I think I want to be called the Commercial and Imperial Revolution Age. Plus the Industrial Revolution Age from 1770 to 1870 sees the pace of technological change amplify. It sees average human living standards start to rise. No longer is better technology just sufficient to counterbalance the smaller farm sizes and less abundant raw materials per worker that come with population growth.

By 1870, the technologies of steampower, textile machinery, metallurgy, and company had brought us to a world where average productivity was maybe 40% higher than it had been in the agrarian age. Of course, the fruits of this productivity were vastly unequally distributed. Hence John Stuart Mill’s inability to see any benefit for the working class.

But after 1870 humanity passes over into a new major watershed: We see the Modern Economic Growth Age begin. Humanity’s technology and organization deployed for productivity in the economy after 1870 grows. It grows at about 2% per year:

- 4 1/2 times as fast as the old 0.45% per year of the 1770-1870 Industrial Revolution Age.
- 13 times as fast as the old 0.15% per year of the Commercial & Imperial Age.
- Fully 60 times as fast as in the Agrarian Age.

Think of it this way: When we try to construct quantitative measures of human technological and organizational progress,we find that the technologies and organizations humanity will deploy in 2031 Will be as far advanced, proportionately, above those of this year 2021 as those of 1500 were advanced beyond those of 890.

And with modern economic growth comes the demographic transition: first the population explosion, and now our current rapid Approach to zero population growth.

And with the coming of modern economic growth comes what we call globalization.

And with the coming of modern economic growth comes an extraordinarily upward leap in the degree of income inequality in the global income distribution. Instead of the income-based distribution of social power becomes no longer a hill, on which some stand higher on it than others. It becomes a cliff, with some at the top, the bottom billion at the bottom, and the others trying to scramble their way up by hand and foot holes.

And with the coming of modern economic growth, we have a period of a century and a half of a American economic ascendancy—an era in which the United States is, in the words of Russian revolutionary Leon Trotsky, “the furnace in which the future is being forged“ Dash an error that, I think, is now at an end. The pole of hardware innovation is mostly in Shenzhen right now, with another hot spot in Taiwan at TSMC, and perhaps still in California with the designers of Apple Silicon. The leading edge of the transition away from carbon energy is in many places, but not in the U.S.

That is the big-picture sweep look at economic growth.

2168 words

2. Economists' Growth Theories

2.1. The Basics of the Solow Growth Model

2.1.1. Preliminaries

2.1.1.1. The Production Function

The first behavioral relationship in the Solow Growth Model is the **production function**: the relationship between the economy's level of income and production Y and its three determinants: the **labor force** L, the **efficiency of labor** E, and the economy's **capital-intensity** κ, which is defined as and measured by the quotient of the economy's **capital stock** K and its level of **total income and production** Y.

$$\kappa = \frac{K}{Y}$$

Satisfy Three Rules of Thumb: The Solow growth model requires that this behavioral relationship satisfy three rules of thumb:

1. A proportional increase in the economy's capital intensity $\kappa=K/Y$, measured by the capital stock divided by total income, will carry with it the same proportional increase in total income and production no matter how rich and productive the economy is. A 1% increase in capital intensity will always increase income and production by the same proportional amount.

2. If two economies have the same capital intensity, defined as the same capital-output ratio κ, and have the same level of technology- and organization-driven efficiency-of-labor E, then the ratio of their levels of income and output will be equal to the ratio of their labor forces L.
3. If two economies have the same capital intensity, defined as the same capital-output ratio κ, and have the same labor forces, then the ratio of their levels of income and output will be equal to the ratio of their technology- and organization-driven efficiencies-of-labor E.

.

There is one and only one way to write an algebraic expression that satisfies these two rule-of-thumb conditions. It is:

$$Y = \kappa^{\theta} E L$$

We write a lower-case y to stand for income and production per worker, and so:

$$y = \kappa^{\theta} E$$

Note: You might object that this is circular: production and total income Y depend on capital intensity κ defined as the capital-output ratio K/Y, but how can you calculate κ and thus calculate Y when you need to know Y to calculate κ? If this worries you, you could start elsewhere, with a different-looking production function:

$$Y = K^{\alpha}(EL)^{1-\alpha}$$

Then divide both sids by Y^{α}, and do some algebra:

$$\frac{Y}{Y^{\alpha}} = \frac{K^{\alpha}}{Y^{\alpha}}(EL)^{1-\alpha}$$
$$Y^{1-\alpha} = \left(\frac{K}{Y}\right)^{\alpha}(EL)^{1-\alpha}$$
$$Y^{1-\alpha} = \kappa^{\alpha}(EL)^{1-\alpha}$$

$$Y^{\left(\frac{1-\alpha}{1-\alpha}\right)} = \kappa^{\left(\frac{\alpha}{1-\alpha}\right)} (EL)^{\left(\frac{1-\alpha}{1-\alpha}\right)}$$

$$Y = \kappa^{\left(\frac{\alpha}{1-\alpha}\right)} EL$$

Then define $\alpha = \theta/(1+\theta)$ and thus $\theta = \alpha/(1+ \alpha)$. That gets us exactly where we wanted to go.

For historical reasons, almost all presentations of the Solow growth model use the forms with α rather than the forms with θ. This adds complexity, to no benefit that I see. We will work with the θ-forms.

Why These Rules of Thumb?: Why did Robert Solow back in 1956 look for an algebraic formula for his production function that would satisfy these three rules-of-thumb? Because economists like to analyze a situation by simplifying it, ruthlessly, when they can get away with it.

Rule-of-thumb (1) is thus a simplifying assumption: an intellectual bet that the process of aggregate economic growth is likely to look very similar as an economy goes from an income-per-capita level of 10,000 to 20,000 dollars per worker per year as when it goes from 40,000 to 80,000 dollars per worker per year. It is worth making only as long as that is in fact true—that the similarities in the aggregate overall economic growth process in different decades and at different income-per-worker levels outweigh the differences. If that were not or were to cease being the case, we should drop that rule-of-thumb assumption. So far, so good.

Rule-of-thumb (2) is simply that holding other things—capital intensity and efficiency-of-labor—constant, you can always duplicate what you are doing and produce and earn twice as much. Again, it is worth making only as long as that is in fact true, or approximately true. Again, so far, so good.

Rule-of-thumb (3) is a recognition that better-organized economies making better use of technology will be more productive: it is best thought of as a definition of how we are going to construct our quantitative index of the level of applied technological knowledge combined with efficient economic organization—the variable E that we call the efficiency-of-labor. It has no deeper implications.

2.1.1.2. Investment and Capital Accumulation

Following economists' custom of ruthless simplification, assume that individuals, households, and businesses desire to save a fraction s of their gross income Y, so that total savings are:

$$S=sY$$

We call s the economy's saving-investment rate (to remind us that s is measuring both the flow of saving into the economy's financial markets and also the share of total production that is invested and used to build up and increase the economy's capital stock). Assume that there are no problems in translating individuals', households', and businesses' desires to save some of their income Y into investment I:

$$I=S=sY$$

We typically assume that s is constant. We do, however, think about the consequences of its taking a permanent upward or downward jump at some particular moment of time. The background assumption, however—made because it makes formulas much simpler—will always be that s will then remain at its jumped-to value as far as we look into the future.

While the saving-investment rate s is assumed constant in the basic Solow model, the economy's capital stock K is not. It changes from year to year from investment and also from depreciation:

$$\frac{dK}{dt} = I - \Delta$$

Assume that:

$$\Delta = \delta K$$

Each year a fraction δ of the existing capital stock depreciates and wears out, so that the rate of change of the capital stock is:

$$\frac{dK}{dt} = I - \delta K$$

The growth of the economy's capital stock K is thus determined by investment, a share s of income Y, minus depreciation, a share δ of the current capital stock K/

2.1.1.3. The Labor Force and the Efficiency-of-Labor

If the labor force L were constant, and if technological and organizational progress plus educational factorsn that add to the efficiency-of-labor E were constant, we could immediately move on. But the economy's labor force L grows, as more people turn 18 or so and join the labor force than retire, and as immigrants continue to arrive. The efficiency of labor E rises as science and technology progress, as people keep thinking of new and more efficient forms of business organization, and people go to school and learn on the job.

We assume—once again making a simplifying leap—that the economy's labor force L's proportional rate g_L is a constant rate *n*. Note that this *n* is not the same across economies or scenarios. Since we want to tackle simple cases first, our background assumption will be that *n* is constant now as far as we can see into the future. But we will drop and vary this when we want to.

Thus between this year and the next the labor force grows according to the formula:

$$\frac{dL}{dt} = g_L L = nL$$

Next year's labor force *L* will thus be a fraction *n* higher than this year's labor force.

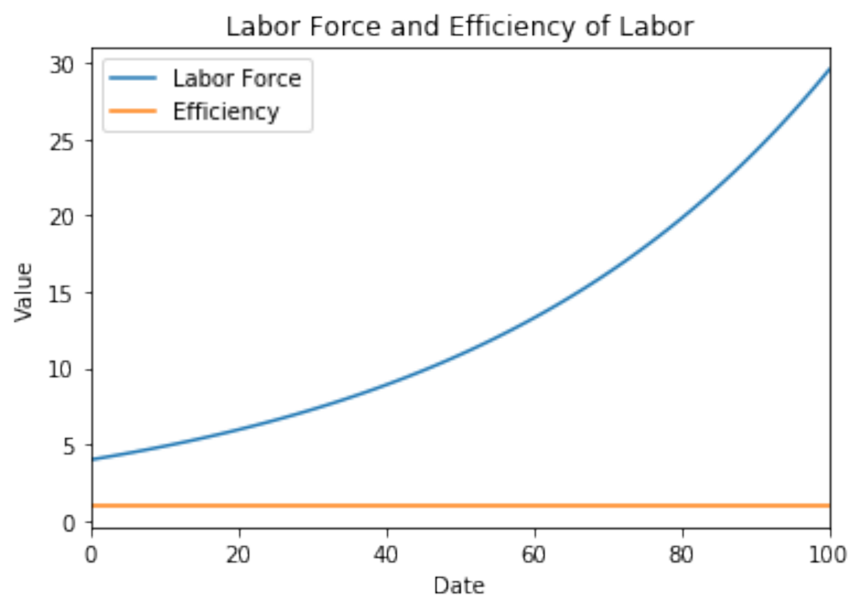
We also assume—once again making a simplifying leap—that the economy's efficiency of labor *E*'s proportional growth rate *g* is a constant every year. Note that g is not the same across countries or scenarios. Note that it can and does shift over time in any one country. But we want to tackle simple cases first. A constant efficiency-of-labor growth rate *g* is simple. Thus our background assumption will be that *g* is constant as far as we can see into the future.

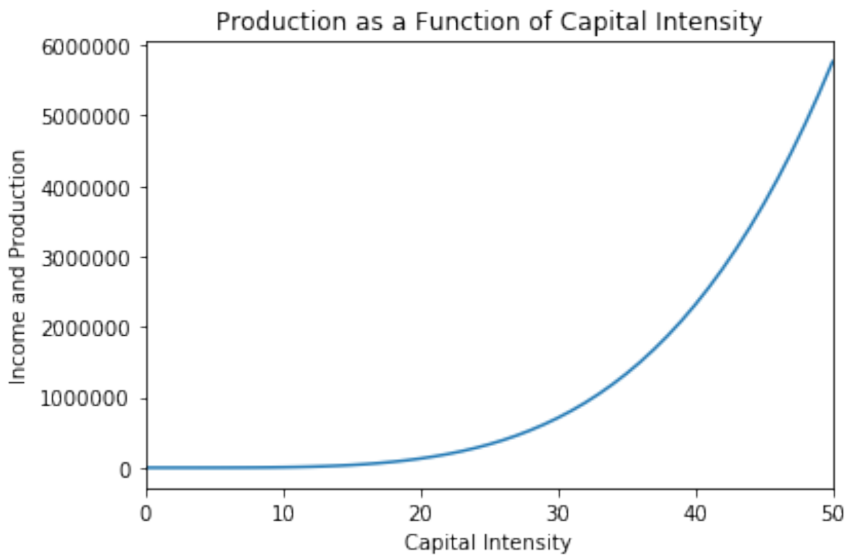
Then between this year and the next the efficiency of labor grows according to the formula:

$$\frac{dE}{dt} = gE$$

2.1.1.4. Gaining Intuition for n, g, L, E, Y,

What does it mean to say that the proportional growth rate of the labor force L is n=0.02—2% per year? It means that the labor force would double every thirty-five years. What does it mean to say that labor efficiency E's proportional growth rate is g=0.05 —5% per year? It means that labor efficiency doubles every twelve years. What are the implications of θ=1? It means that if you were to compare two economies with no differences save that the capital-intensity κ of one were twice that of the other, output and output per worker in the first would also be twice as great as in the other. What are the implications of θ=3 ? It means that if you were to compare two economies with no differences save that the capital-intensity κ of one were twice that of the other, output and output per worker in the first would also be nine times as great as in the other.





Production as a Function of Capital Intensity
Income and Production
Capital Intensity
6000000
5000000
4000000
3000000
2000000
1000000
0
0
10
20
30
40
50

2.2. The Solow Equilibrium Condition

2.2.1. Balanced Growth

Multiply the economy's capital-intensity κ by the economy's level of total income and production, and you get the economy's capital stock K:

$$K = \kappa Y$$

the amount of produced means of production that the economy has inherited from its past.

Now take the natural log of and then take the time derivative of the production function:

$$Y = \kappa^{\theta} E L$$

The result is

$$\ln(Y) = \theta \ln(\kappa) + \ln(L) + \ln(E)$$

0r:

$$\frac{1}{Y}\frac{dY}{dt} = g_Y = \theta\left(\frac{1}{\kappa}\frac{d\kappa}{dt}\right) + \frac{1}{L}\frac{dL}{dt} + \frac{1}{E}\frac{dE}{dt}$$

We have assumed that the second term on the right-hand-side is n and that the third term is g. If the capital-intensity κ
κ is constant, then the left-hand-side will be equal to n+g: that will then be the proportional growth rate of income and production Y. If Y is growing at rate $n+g$ and κ is constant, then the economy's capital-stock K will also be growing at $n+g$. Everything will then be in *balanced growth*. And if the economy is in *balanced growth*

it will stay there. And if the economy is not in *balanced growth*, it will over time head for a configuration that is.

2.2.2. When is Capital-Intensity Constant?

We have assumed that a constant fraction s of total income Y was saved and invested to add to the capital stock. We have assumed that a share δ of the capital stock rusts and erodes and disappears each year. Thus we have assumed that the capital-stock K was changing at:

$$\frac{dK}{dt} = sY - \delta K$$

And thus the capital stock was growing at a proportional growth rate:

$$\frac{1}{K}\frac{dK}{dt} = g_K = \frac{s}{\kappa} - \delta$$

We thus can find:

$$\frac{1}{Y}\frac{dY}{dt} = g_Y = \theta\left(\frac{1}{K}\frac{dK}{dt} - \frac{1}{Y}\frac{dY}{dt}\right) + \frac{1}{L}\frac{dL}{dt} + \frac{1}{E}\frac{dE}{dt}$$

$$(1+\theta)\frac{1}{Y}\frac{dY}{dt} = \theta\left(\frac{s}{\kappa} - \delta\right) + n + g$$

So the proportional rate of growth of capital-intensity is

$$\frac{1}{\kappa}\frac{d\kappa}{dt} = g_\kappa = \frac{s}{\kappa} - \delta - \left(\frac{\theta}{1+\theta}\right)\left(\frac{s}{\kappa} - \delta\right) - \frac{n+g}{1+\theta}$$

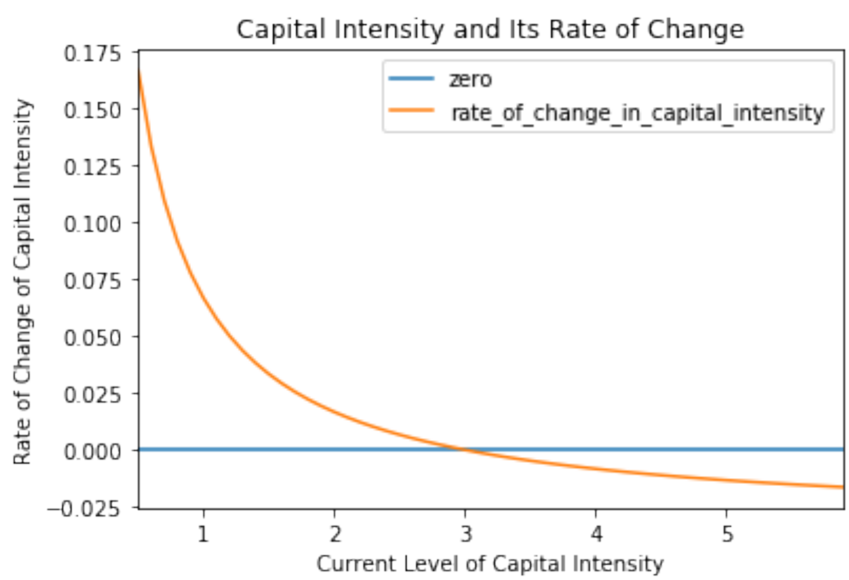
$$\frac{1}{\kappa}\frac{d\kappa}{dt} = \frac{s/\kappa - (n+g+\delta)}{1+\theta}$$

Thus the capital-stock will be growing at the rate $n+g$ required for balanced growth if and only if:

$$\frac{s}{\kappa} - \delta = n + g$$

That tells us that the economy will be growing along a balanced-growth path if and only if:

$$\kappa = \frac{s}{n+g+\delta}$$



2.3. Growing Along & Converging to the Balanced-Growth Equilibrium Path

2.3.1. The Balanced-Growth Equilibrium Path

2.3.1.1. The Balanced-Growth Equilibrium Capital Intensity κ^*

We define $\kappa*$ as that value of capital-intensity κ for which, at the current levels of the parameters n, g, δ, s, and θ, the equation:

$$\frac{s}{\kappa} - \delta = n + g$$

is satisfied. That is true if and only if:

$$\kappa = \kappa^* = \frac{s}{n + g + \delta}$$

If the capital-intensity $\kappa=\kappa*$, then it is constant. The economy is then in balanced growth. The proportional growth rate g_Y of total income and production in the economy is then equal to n+g, the sum of the growth rate of the labor force and the growth rate of the efficiency of labor. The proportional growth rate g_y of income and per worker is then equal to g, the growth rate of the efficiency of labor. The proportional growth rate of the economy's total capital stock is then the same $n+g$ as the growth rate of income and production

2.3.1.2. Calculating the Balanced-Growth Equilibrium Path ¶

We can then—if we know the parameter values of the model, the initial values L_0 and E_0 of the labor force and labor efficiency at some time we index equal to 0, and that the economy is on its balanced-growth equilibrium path—calculate what all variables of interest in the economy will be at any time whatsoever:

Total income and production will be:

$$Y_t^* = (\kappa^*)^\theta E_t L_t = (\kappa^*)^\theta e^{gt} E_0 e^{nt} L_0 = \left(s/(n+g+\delta)\right)^\theta e^{gt} E_0 e^{nt} L_0$$

Income and production per worker will be:

$$y_t^* = (\kappa^*)^\theta E_t = (\kappa^*)^\theta e^{gt} E_0 = \left(s/(n+g+\delta)\right)^\theta e^{gt} E_0$$

The capital stock will be:

$$K_t^* = \kappa^* Y_t^* = \left(s/(n+g+\delta)\right)^{(1+\theta)} e^{gt} E_0 e^{nt} L_0$$

The labor force will be:

$$L_t^* = e^{nt} L_0$$

And labor efficiency will be:

$$E_t^* = e^{gt} E_0$$

2.3.2. Converging to the Balanced-Growth Equilibrium Path

2.3.2.1. The Dynamics of Capital Intensity

But what if $\kappa \neq \kappa*$? What happens then?

Since $s = \kappa*(n+g+\delta)$, we can multiply:

$$\frac{1}{\kappa}\frac{d\kappa}{dt} = \frac{s/\kappa - (n+g+\delta)}{1+\theta}$$

by κ and then rewrite it in terms of the equilibrium capital-intensity $\kappa*$ as:

$$\frac{d\kappa}{dt} = s/(1+\theta) - (n+g+\delta)\kappa/(1+\theta)$$

$$\frac{d\kappa}{dt} = (n+g+\delta)\kappa^*/(1+\theta) - (n+g+\delta)\kappa/(1+\theta)$$

$$\frac{d\kappa}{dt} = -\frac{n+g+\delta}{1+\theta}(\kappa - \kappa^*)$$

This equation always holds at every moment, for that moment's values of n,g,δ,θ, and s, whatever they may be.

This is the very first differential equations one encounters in mathematics. If n,g,δ,θ, and s are constant, this equation has the solution, if the value of capital-intensity κ_0 is known at some time t=0, of:

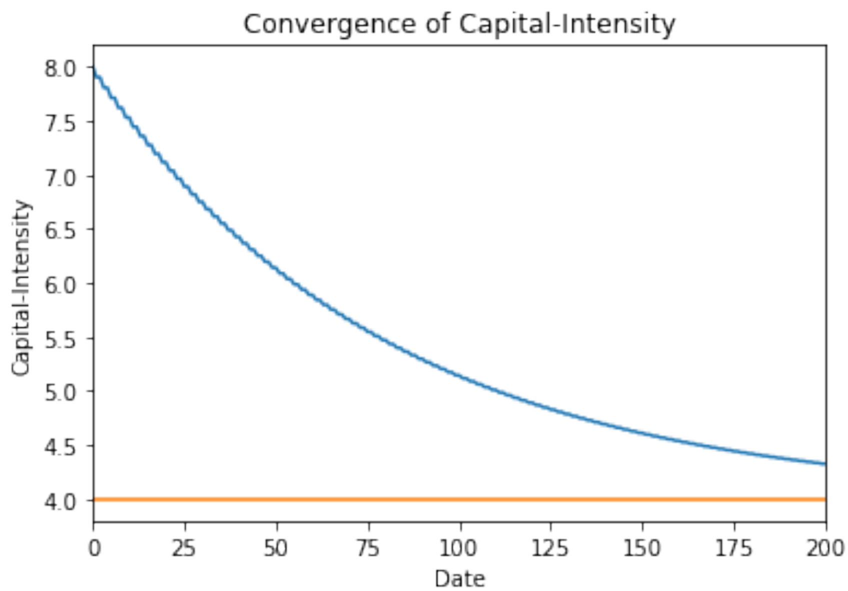
$$\kappa = \kappa^* + e^{-((n+g+\delta)/(1+\theta))t}(\kappa_0 - \kappa^*)$$

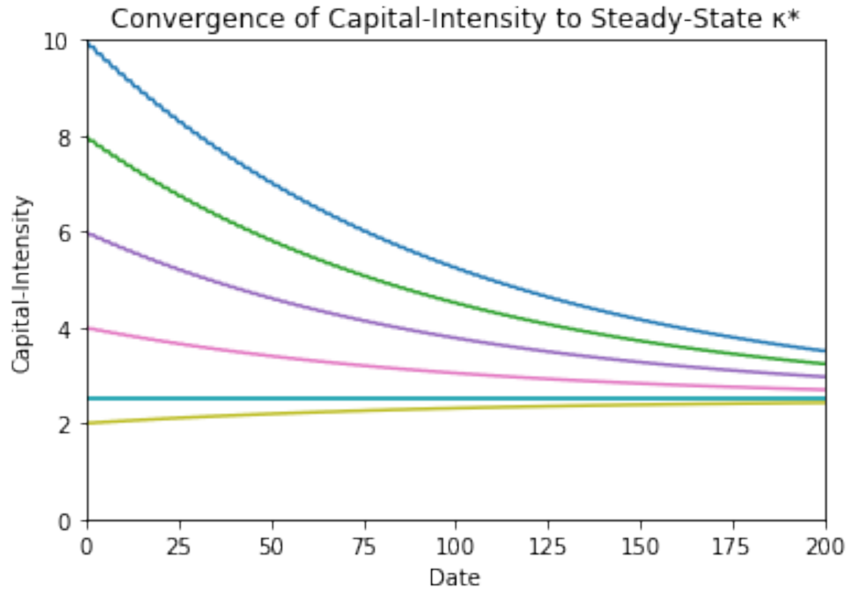
If any of n,g,δ,θ, and s change, you then have to recalibrate and recompute, with a new initial value of κ_0, equal to its value when the model's parameters jumped, and a new and different value of $\kappa*$.

If n,g,δ,θ, and s are constant or near-constant, then these last equations are very powerful tools: they tell us that the economy's capital-intensity κ follows, over time, a path of exponential convergence. κ is, at time zero, equal to its initial condition κ_0. It then converges towards its asymptote $\kappa*$, reducing the gap between its current value and $\kappa*$ to a fraction $1/e$ in the interval between time t and time t + $\Delta_{1/e}t$, where this $1/e$ convergence time is:

$$\Delta_{1/e}t = (n + g + \delta)/(1 + \theta)$$

2.3.2.2. Gaining Intuition About the Convergence of Capital-Intensity to $\kappa*$





2.3.2.3. The Dynamics of the Other Variables in the Economy

There are analogous equations for all the other variables in the economy:

$$Y_t = (\kappa_t)^{\theta} E_t L_t = (\kappa_t)^{\theta} e^{gt} E_0 e^{nt} L_0$$

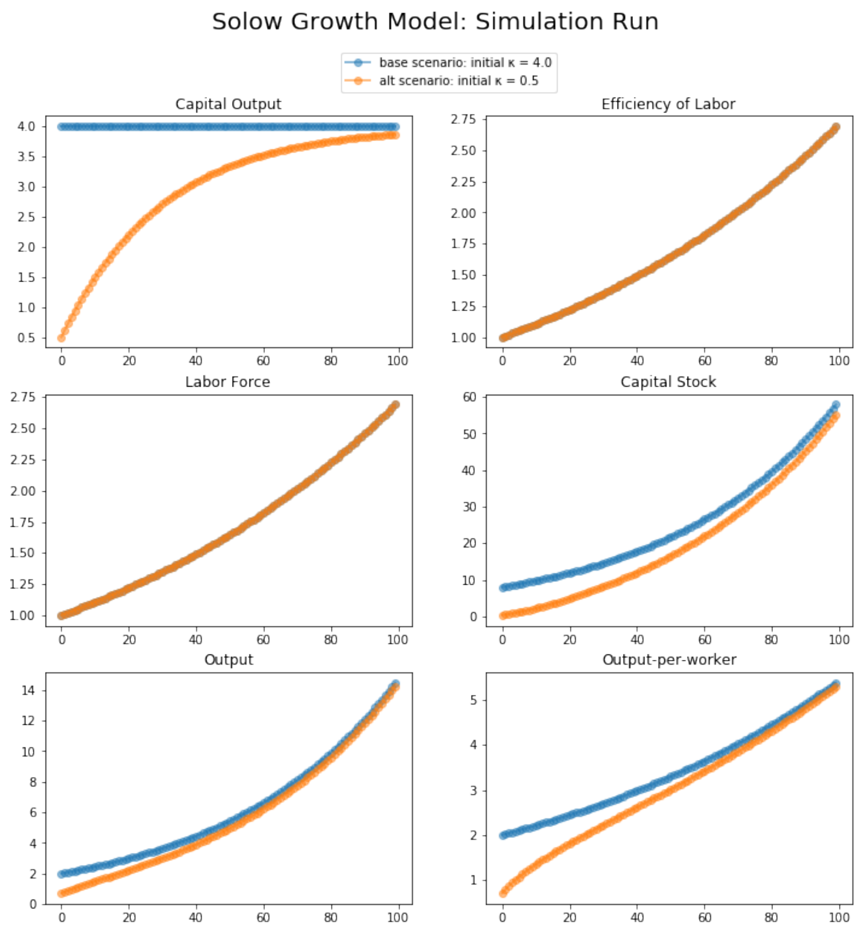
$$y_t = (\kappa_t)^{\theta} E_t = (\kappa_t)^{\theta} e^{gt} E_0$$

$$K_t = \kappa_t Y_t$$

$$L_t = e^{nt} L_0$$

$$E_t = e^{gt} E_0$$

2.3.2.4. Gaining Intuition About Convergence and Shocks

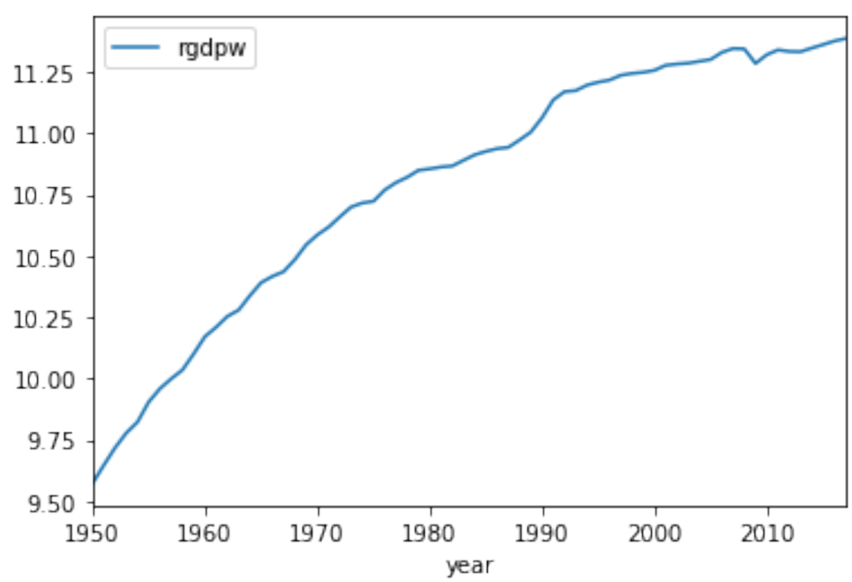


2.4. Using the Solow Growth Model

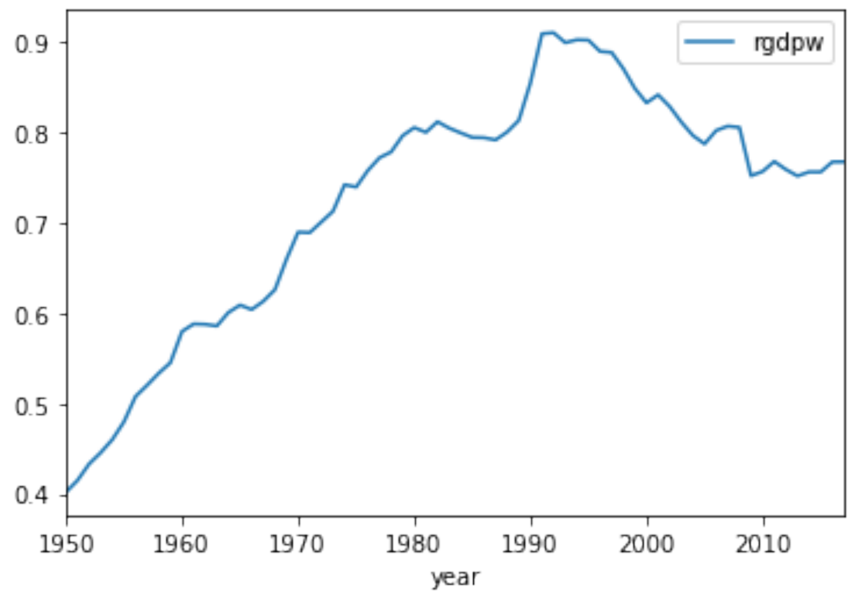
2.4.1. Convergence to the Balanced-Growth Path

2.4.1.1. The Example of Post-WWII West Germany

Economies do converge to and then remain on their balanced-growth paths. The West German economy after World War II is a case in point. The defeat of the Nazis left the German economy at the end of World War II in ruins. Output per worker was less than one-third of its prewar level. The economy's capital stock had been wrecked and devastated by three years of American and British bombing and then by the ground campaigns of the last six months of the war.

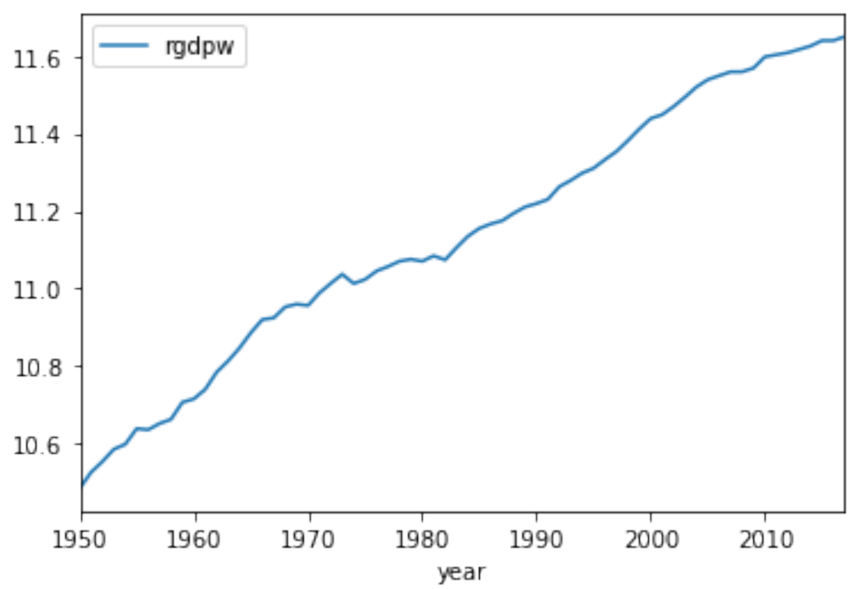


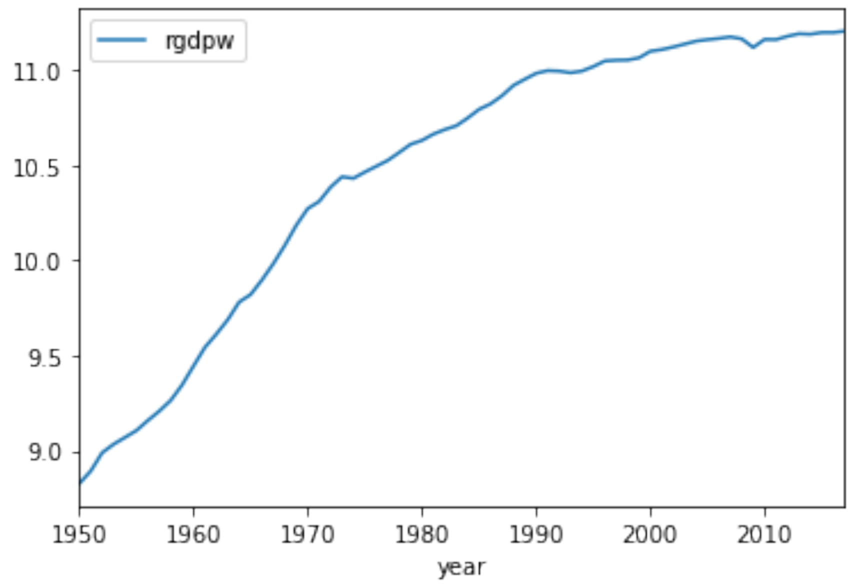
But in the years immediately after the war, the West German economy's capital-output ratio rapidly grew and converged back to its prewar value. Within 12 years the West German economy had closed half the gap back to its pre-World War II growth path. And within 30 years the West German economy had effectively closed the entire gap between where it had started at the end of World War II and its balanced-growth path.



The two figures above show, respectively, the natural logarithm of absolute real national income per worker for the German economy and real national income per worker relative to the U.S. value, both since 1950. By 1980 the German economy had converged: its period of rapid recovery growth was over, and national income per capita then grew at the same rate as that in the U.S., which had not suffered wartime destruction pushing it off and below its steady-state balanced-growth path. Then in 1990, at least according to this set of estimates, the absorption of the formerly communist East German state into the *Bundesrepublik* was an enormous benefit: the expanded division of labor and return of the market economy allowed productivity in the German east to more than double almost overnight. Thereafter the German economy has lost some ground relative to the U.S. as the U.S.'s leading information technology hardware and software sectors have been much stronger leading sectors than Germany's precision machinery and manufacturing sectors.

By comparison, the United States shows no analogous period of rapid growth catching up to a steady-state balanced-growth path. (There is, however, a marked boom in the 1960s, and then a return to early trends in the late 1970s and 1980s, followed by a return to previous normal growth in the 1990s and then a fall-off in growth after 2007.)

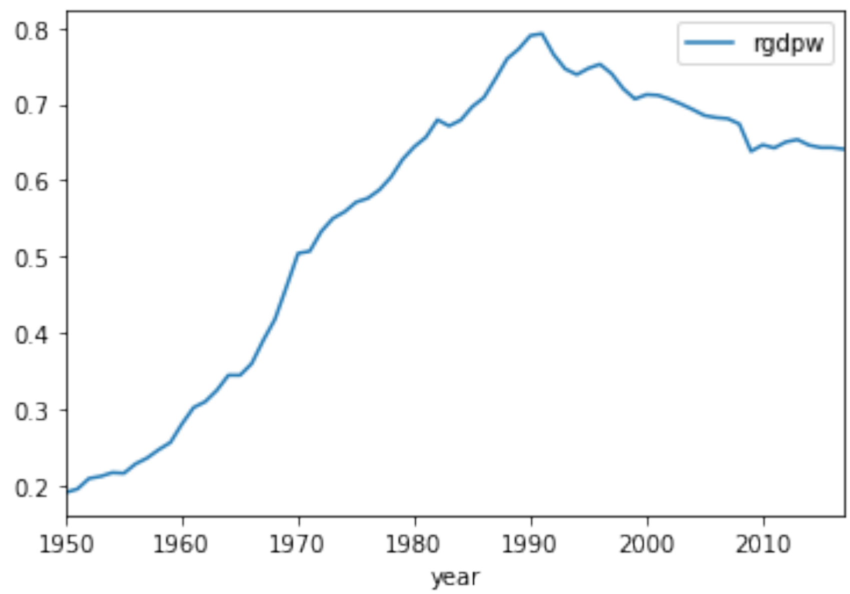




2.4.1.2. The Example of Post-WWII Japan ¶

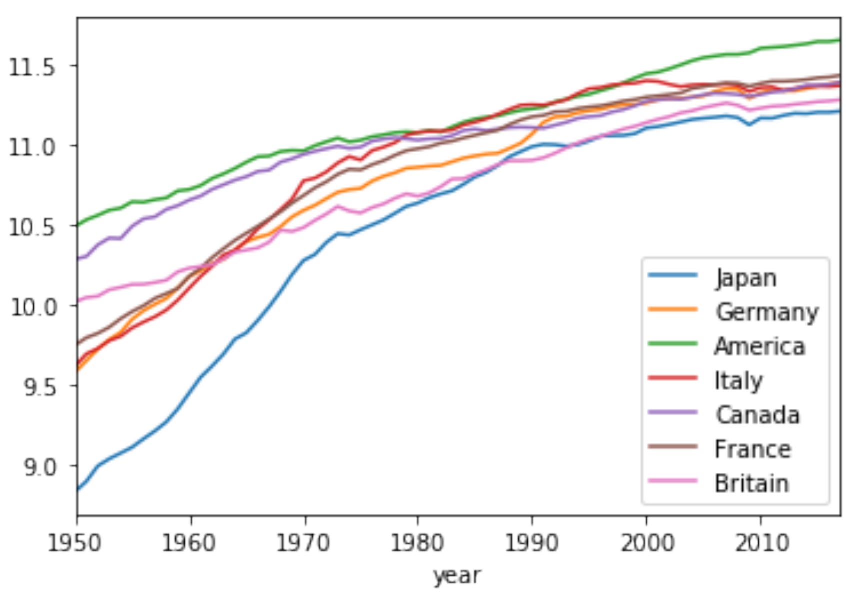
The same story holds in an even stronger form for the other defeated fascist power that surrendered unconditionally to the U.S. at the end of World War II.

In 1950, largely as a result of Curtis LeMay's B-29s, Japan is only half as productive as Germany, and only one-fifth as productive as the United States. Once again, it converges rapidly. After 1990 Japan no longer grows faster than and catches up to the United States. Indeed, like Germany it thereafter loses ground as its world class manufacturing sectors are also less powerful leading sectors than the United States's information technology hardware and software complexes.

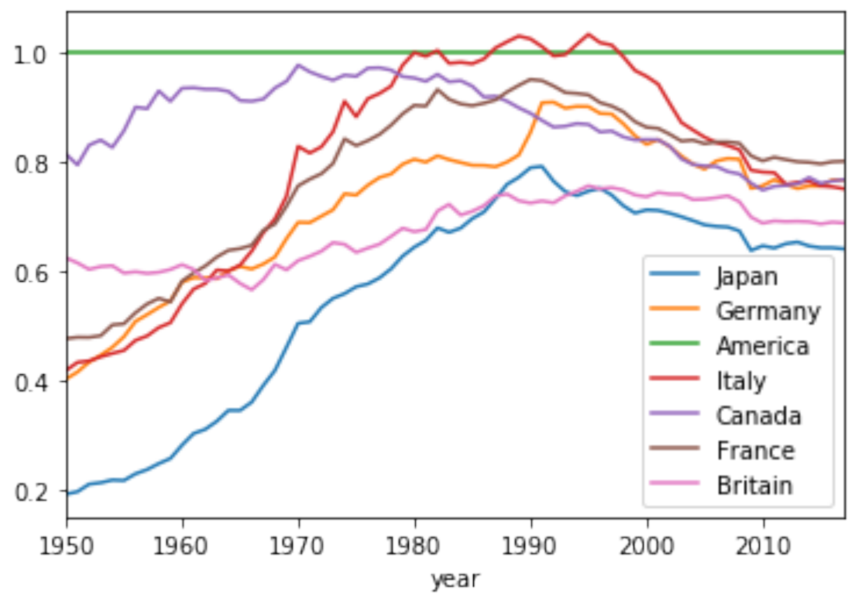


2.4.1.3. The Post-WWII G-7

The same story holds for the other members of the G-7 group of large advanced industrial economies as well.



The idea—derived from the Solow model—that economi,es pushed off and below their steady-state balanced-growth paths by the destruction and chaos of war thereafter experience a period of supergrowth that ebbs as they approach their steady-state balanced-growth paths from below story holds for the other members of the G-7 group of large advanced industrial economies as well. In increasing order of the magnitude of their shortfall vis-a-vis the U.S. and the speed of recovery supergrowth, we have: France, Italy, Germany, and Japan. The three economies that escaped wartime chaos and destruction—the U.S., Britain, and Canada—do not exhibit supergrowth until catchup to their steady-state balanced-growth paths.



There is a lot more going on in the post-WWII history of the G-7 economies than just catchup to their steady-state balanced-growth paths after the destruction of World War II: Why do the other economies lose ground vis-a-vis the U.S. after 1990? Why does the U.S. exhibit a small speedup, slowdown, speedup, and then renewed slowdown again? What is it with Britain's steady-state balanced-growth path having so much lower productivity than the other Europeans? Why is Japan the most different from its G-7 partners? And what is it with Italy's attaining U.S. worker productivity levels in 1980, and then its post-2000 relative collapse? (The post-2000 collapse in Italian growth is real; the estimate that it was as productive as the U.S. from 1980-2000 is a data construction error.)

2.4.2. Analyzing Jumps in Parameter Values

What if one or more of the parameters in the Solow growth model were to suddenly and substantially shift? What if the labor-force growth rate were to rise, or the rate of technological progress to fall?

One principal use of the Solow growth model is to analyze questions like these: how changes in the economic environment and in economic policy will affect an economy's long-run levels and growth path of output per worker Y/L.

Let's consider, as examples, several such shifts: an increase in the growth rate of the labor force n, a change in the economy's saving-investment rate s, and a change in the growth rate of labor efficiency g. All of these will have effects on the balanced- growth path level of output per worker. But only one—the change in the growth rate of labor efficiency—will permanently affect the growth rate of the economy.

We will assume that the economy starts on its balanced growth path—the old balanced growth path, the pre-shift balanced growth path. Then we will have one (or more) of the parameters—the savings-investment rate s, the labor force growth rate n, the labor efficiency growth rate g—jump discontinuously, and then remain at its new level indefinitely. The jump will shift the balanced growth path. But the level of output per worker will not immediately jump. Instead, the economy's variables will then, starting from their old balanced growth path values, begin to converge to the new balanced growth path—and converge in the standard way.

Remind yourselves of the key equations for understanding the model:

The level of output per worker is:

$$\frac{Y}{L} = \left(\frac{K}{Y}\right)^{\theta} E$$

The balanced-growth path level of output per worker is:

$$\left(\frac{Y}{L}\right)^{*} = \left(\frac{s}{n+g+\delta}\right)^{\theta} E$$

The speed of convergence of the capital-output ratio to its balanced-growth path value is:

$$\frac{d(K/Y)}{dt} = -(1-\alpha)(n+g+\delta)\left[\frac{K}{Y} - \frac{s}{(n+g+\delta)}\right]$$

where (you recall):

$$\theta = \alpha/(1-\alpha)$$

and:

$$\alpha = \theta/(1 + \theta)$$

2.4.2.1. A Shift in the Labor-Force Growth Rate

Real-world economies exhibit profound shifts in labor-force growth. The average woman in India today has only half the number of children that the average woman in India had only half a century ago. The U.S. labor force in the early eighteenth century grew at nearly 3 percent per year, doubling every 24 years. Today the U.S. labor force grows at 1 percent per year. Changes in the level of prosperity, changes in the freedom of migration, changes in the status of women that open up new categories of jobs to them (Supreme Court Justice Sandra Day O'Connor could not get a private-sector legal job in San Francisco when she graduated from Stanford Law School even with her amazingly high class rank), changes in the average age of marriage or the availability of birth control that change fertility—all of these have powerful effects on economies' rates of labor-force growth.

What effects do such changes have on output per worker Y/L—on our mea sure of material prosperity? The faster the growth rate of the labor force n, the lower will be the economy's balanced-growth capital-output ratio $s/(n + g - \delta)$. Why? Because each new worker who joins the labor force must be equipped with enough capital to be productive and to, on average, match the productivity of his or her peers. The faster the rate of growth of the labor force, the larger the share of current investment that must go to equip new members of the labor force with the capital they need to be productive. Thus the lower will be the amount of investment that can be devoted to building up the average ratio of capital to output.

A sudden and permanent increase in the rate of growth of the labor force will lower the level of output per worker on the balanced-growth path. How large will the long-run change in the level of output be, relative to what would have happened had labor-force growth not increased? It is straightforward to calculate if we know the other parameter values, as is shown in the example below.

An Example: An Increase in the Labor Force Growth Rate: Consider an economy in which the parameter α is 1/2, the efficiency of labor growth rate g is 1.5 percent per year, the depreciation rate δ is 3.5 percent per year, and the saving rate s is 21 percent. Suppose that the labor-force growth rate suddenly and permanently increases from 1 to 2 percent per year.
Before the increase in the labor-force growth rate, in the initial steady-state, the balanced-growth equilibrium capital-output ratio was:

$$\left(\frac{K_{in}}{Y_{in}}\right)^{*} = \frac{s_{in}}{(n_{in} + g_{in} + \delta_{in})} = \frac{0.21}{(0.01 + 0.015 + 0.035)} = \frac{0.21}{0.06} = 3.5$$

(with subscripts "in" for "initial).

After the increase in the labor-force growth rate, in the alternative steady state, the new balanced-growth equilibrium capital-output ratio will be:

$$\left(\frac{K_{alt}}{Y_{alt}}\right)^{*} = \frac{s_{alt}}{(n_{alt} + g_{alt} + \delta_{alt})} = \frac{0.21}{(0.02 + 0.015 + 0.035)} = \frac{0.21}{0.07} = 3$$

(with subscripts "alt" for "alternative").

Before the increase in labor-force growth, the level of output per worker along the balanced-growth path was equal to:

$$\left(\frac{Y_{t,in}}{L_{t,in}}\right)^* = \left(\frac{s_{in}}{(n_{in} + g_{in} + \delta_{in})}\right)^{\alpha/(1-\alpha)} E_{t,in} = 3.5E_{t,in}$$

After the increase in labor-force growth, the level of output per worker along the balanced-growth path will be equal to:

$$\left(\frac{Y_{t,alt}}{L_{t,alt}}\right)^* = \left(\frac{s_{alt}}{(n_{alt} + g_{alt} + \delta_{alt})}\right)^{\alpha/(1-\alpha)} E_{t,alt} = 3E_{t,alt}$$

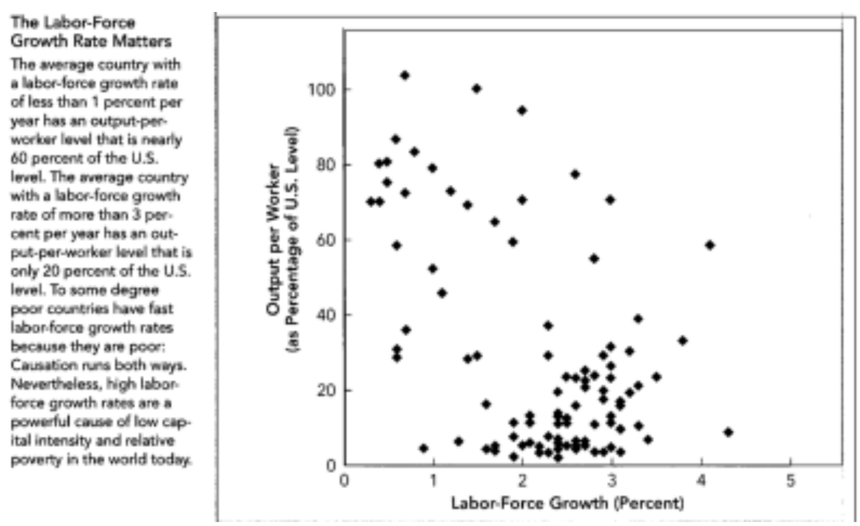
This fall in the balanced-growth path level of output per worker means that in the long run—after the economy has converged to its new balanced-growth path—one-seventh of its per worker economic prosperity has been lost because of the increase in the rate of labor-force growth.

In the short run of a year or two, however, such an increase in the labor-force growth rate has little effect on output per worker. In the months and years after labor-force growth increases, the increased rate of labor-force growth has had no time to affect the economy's capital-output ratio. But over decades and generations, the capital-output ratio will fall as it converges to its new balanced-growth equilibrium level.

A sudden and permanent change in the rate of growth of the labor force will immediately and substantially change the level of output per worker along the economy's balanced-growth path: It will shift the balanced-growth path for output per worker up (if labor-force growth falls) or down (if labor-force growth rises). But there is no corresponding immediate jump in the actual level of output per worker in the economy. Output per worker doesn't immediately jump—it is just that the shift in the balanced-growth path means that the economy is no longer in its Solow growth model long-run equilibrium.

Empirics: The Labor-Force Growth Rate Matters: The average country with a labor-force growth rate of less than 1 percent per year has an output-per-worker level that is nearly 60 percent of the U.S. level. The average country with a labor-force growth rate of more than 3 percent per year has an output-per-worker level that is only 20 percent of the U.S. level.
To some degree poor countries have fast labor-force growth rates because they are poor: Causation runs both ways. Nevertheless, high labor-force growth rates are a powerful cause of low capital intensity and relative poverty in the world today.

The Labor Force Growth Rate Matters: Output per Worker and Labor Force Growth



How important is all this in the real world? Does a high rate of labor-force growth play a role in making countries relatively poor not just in economists' models but in reality? It turns out that it is important. Of the 22 countries in the world in 2000 with output-per-worker levels at least half of the U.S. level, 18 had labor-force growth rates of less than 2 percent per year, and 12 had labor-force growth rates of less than 1 percent per year. The additional investment requirements imposed by rapid labor-force growth are a powerful reducer of capital intensity and a powerful obstacle to rapid economic growth.

It takes time, decades and generations, for the economy to converge to its new balanced-growth path equilibrium, and thus for the shift in labor-force growth to affect average prosperity and living standards. But the time needed is reason for governments that value their countries' long-run prosperity to take steps now (or even sooner) to start assisting the demographic transition to low levels of population growth. Female education, social changes that provide women with more opportunities than being a housewife, inexpensive birth control—all these pay large long-run dividends as far as national prosperity levels are concerned.

U.S. President John F Kennedy used to tell a story of a retired French general, Marshal Lyautey, "who once asked his gardener to plant a tree. The gardener objected that the tree was slow-growing and would not reach maturity for a hundred years. The Marshal replied, 'In that case, there is no time to lose, plant it this afternoon.'"

2.4.2.2. The Algebra of a Higher Labor Force Growth Rate

But rather than calculating example by example, set of parameter values by set of parameter values, we can gain some insight by resorting to algebra, and consider in generality the effect on capital-output ratios and output per worker levels of an increase Δn in the labor force growth rate, following an old math convention of using "Δ" to stand for a sudden and discrete change.

Assume the economy has its Solow growth parameters, and its initial balanced-growth path capital-output ratio:

$$\left(\frac{K_{in}}{Y_{in}}\right)^* = \frac{s_{in}}{(n_{in} + g_{in} + \delta_{in})}$$

with "in" standing for “initial".

And now let us consider an alternative scenario, with "alt" standing for "alternative", in which things had been different for a long time:

$$\left(\frac{K_{alt}}{Y_{alt}}\right)^* = \frac{s_{alt}}{(n_{alt} + g_{alt} + \delta_{alt})}$$

For the g and δ parameters, their initial values are their alternative values. And for the labor force growth rate:

$$n_{alt} = n_{in} + \Delta n$$

So we can then rewrite:

$$\left(\frac{K_{alt}}{Y_{alt}}\right)^* = \frac{s_{in}}{(n_{in} + g_{in} + \delta_{in})} \frac{(n_{in} + g_{in} + \delta_{in})}{(n_{in} + \Delta n + g_{in} + \delta_{in})} = \frac{s_{in}}{(n_{in} + g_{in} + \delta_{in})} \left[\frac{1}{1 + \frac{\Delta n}{(n_{in} + g_{in} + \delta_{in})}}\right]$$

The first term on the right hand side is just the initial capital-output ratio, and we know that 1/(1+x) is approximately 1−x for small values of x, so we can make an approximation:

$$\left(\frac{K_{alt}}{Y_{alt}}\right)^* = \left(\frac{K_{in}}{Y_{in}}\right)^* \left[1 - \frac{\Delta n}{(n_{in} + g_{in} + \delta_{in})}\right]$$

Take the proportional change in the denominator $(n+g+\delta)$ of the expression for the balanced-growth capital-output ratio. Multiply that proportional change by the initial balanced-growth capital-output ratio. That is the differential we are looking for.

And by amplifying or damping that change by raising to the $\alpha/(1-\alpha)$ power, we get the differential for output per worker.

2.4.2.3. A Shift in the Growth Rate of the Efficiency of Labor ¶

Efficiency of Labor the Master Key to Long Run Growth: By far the most important impact on an economy's balanced-growth path values of output per worker, however, is from shifts in the growth rate of the efficiency of labor g. We already know that growth in the efficiency of labor is absolutely essential for sustained growth in output per worker and that changes in g are the only things that cause permanent changes in growth rates that cumulate indefinitely.

Recall yet one more time the capital-output ratio form of the production function:

$$\frac{Y}{L} = \left(\frac{K}{Y}\right)^{\theta} E$$

Consider what this tells us. We know that a Solow growth model economy converges to a balanced-growth path. We know that the capital-output ratio K/Y is constant along the balanced-growth path. We know that the returns-to-investment parameter α is constant. And so the balanced-growth path level of output per worker Y/L grows only if, and grows only as fast as, the efficiency of labor E grows.

Efficiency of Labor Growth and the Capital-Output Ratio: Yet when we took a look at the math of an economy on its balanced growth path:

$$\left(\frac{Y}{L}\right)^* = \left(\frac{s}{n + g + \delta}\right)^\theta E$$

we also see that an increase in g raises the denominator of the first term on the right hand side—and so pushes the balanced-growth capital output ratio down. That implies that the balanced-growth path level of output per worker associated with any level of the efficiency of labor down as well.

It is indeed the case that—just as in the case of an increased labor force growth rate n—an increased efficiency-of-labor growth rate g reduces the economy's balanced-growth capital-output ratio s/(n + g - δ). Why? Because, analogously with an increase in the labor force, increases in the efficiency of labor allow each worker to do the work of more, but they need the machines and buildings to do them. The faster the rate of growth of the efficiency of la or, the larger the share of current investment that must go to keep up with the rising efficiency of old members of the labor force and supply them with the capital they need to be productive. Thus the lower will be the amount of investment that can be devoted to building up or maintaining the average ratio of capital to output.

2.4.3.4. The Algebra of Shifting the Efficiency-of-Labor Growth Rate

The arithmetic and algebra are, for the beginning and the middle, the same as they were for an increase in the rate of labor force growth:

Assume the economy has its Solow growth parameters, and its initial balanced-growth path capital-output ratio:

$$\left(\frac{K_{in}}{Y_{in}}\right)^* = \frac{s}{(n_+ g_{in} + \delta)}$$

(with "in" standing for "initial"). Also consider an alternative scenario, with "alt" standing for "alternative", in which things had been different for a long time, with a higher efficiency-of-labor growth rate g+Δg since some time t=0 now far in the past:

$$\left(\frac{K_{alt}}{Y_{alt}}\right)^* = \frac{s}{(n + g + \Delta g + \delta)}$$

We can rewrite this as:

$$\left(\frac{K_{alt}}{Y_{alt}}\right)^* = \frac{s}{(n + g_{in} + \delta)} \frac{(n + g_{in} + \delta)}{(n + g_{in} + \Delta g + \delta)} = \frac{s}{(n + g_{in} + \delta)} \left[\frac{1}{1 + \frac{\Delta g}{(n + g_{in} + \delta)}}\right]$$

Once again, the first term on the right hand side is just the initial capital-output ratio, and we know that 1/1+x is approximately 1−x for small values of x, so we can once again make an approximation:

$$\left(\frac{K_{alt}}{Y_{alt}}\right)^* = \left(\frac{K_{in}}{Y_{in}}\right)^* \left[1 - \frac{\Delta g}{(n + g_{in} + \delta)}\right]$$

Take the proportional change in the denominator of the expression for the balanced-growth capital output ratio. Multiply that proportional change by the initial balanced-growth capital-output ratio. That is the differential in the balanced-growth capital-output ratio that we are looking for.

But how do we translate that into a differential for output per worker? In the case of an increase in the labor force growth rate, it was simply by amplifying or damping the change in the balanced-growth capital-output ratio by raising it to the power θ=(α/(1−α)) in order to get the differential for output per worker. We could do that because the efficiency-of-labor at every time t E_t was the same in both the initial and the alternative scenarios.

That is not the case here.

Here, the efficiency of labor was the same in the initial and alternative scenarios back at time 0, now long ago. Since then E has been growing at its rate g in the initial scenario, and at its rate g+Δg in the alternative scenario, and so the time subscripts will be important. Thus for the alternative scenario:

$$\left(\frac{Y_{t,alt}}{L_{t,alt}}\right)^{*} \left(\frac{s}{(n+g_{in}+\Delta g+\delta)}\right)^{\theta)} (1+(g_{in}+\Delta g))^{t} E_0$$

while for the initial scenario:

$$\left(\frac{Y_{t,ini}}{L_{t,ini}}\right)^{*} \left(\frac{s}{(n+g_{in}+\delta)}\right)^{\theta} (1+g_{in})^{t} E_0$$

Now divide to get the ratio of output per worker under the alternative and initial scenarios:

$$\left(\frac{Y_{t,alt}/L_{t,alt}}{Y_{t,ini}/L_{t,ini}}\right)^{*} = \left(\frac{n+g_{in}+\delta}{(n+g_{in}+\Delta g+\delta)}\right)^{\theta} (1+\Delta g)^{t}$$

Thus we see that in the long run, as the second term on the right hand side compounds as t grows, balanced-growth path output per worker under the alternative becomes first larger and then immensely larger than output per worker under the initial scenario. Yes, the balanced-growth path capital-output ratio is lower. But the efficiency of labor at any time t is higher, and then vastly higher if Δg_t has had a chance to mount up and thus $(1+\Delta g)^t$ has had a chance to compound.

Yes, a positive in the efficiency of labor growth g does reduce the economy's balanced-growth path capital-output ratio. But these effects are overwhelmed by the more direct effect of a larger g on output per worker. It is the economy with a high rate of efficiency of labor force growth g that becomes by far the richest over time. This is our most important conclusion. In the very longest run, the growth rate of the standard of living—of output per worker—can change if and only if the growth rate of labor efficiency changes. Other factors—a higher saving-investment rate, lower labor-force growth rate, or lower depreciation rate—can and down. But their effects are short and medium effects: They do not permanently change the growth rate of output per worker, because after the economy has converged to its balanced growth path the only determinant of the growth rate of output per worker is the growth rate of labor efficiency: both are equal to g.

Thus, if we are to increase the rate of growth of the standard of living permanently, we must pursue policies that increase the rate at which labor efficiency grows—policies that enhance technological and organizational progress, improve worker skills, and add to worker education.

An Example: Shifting the Growth Rate of the Efficiency of Labor: What are the effects of an increase in the rate of growth of the efficiency of labor? Let's work through an example:

Suppose we have, at some moment we will label time 0, t=0, an economy on its balanced growth path with a savings rate s of 20% per year, a labor force growth rate n or 1% per year, a depreciation rate δ of 3% per year, an efficiency-of-labor growth rate g of 1% per year, and a production function curvature parameter α of 1/2 and thus a θ=1. Suppose that at that moment t=0 the labor force L_0 is 150 million, and the efficiency of labor E_0 is 35000.

It is straightforward to calculate the economy at that time 0. Because the economy is on its balanced growth path, its capital-output ratio K/Y is equal to the balanced-growth path capital-output ratio (K/Y)*:

$$\frac{K_0}{Y_0} = \left(\frac{K}{Y}\right)^* = \frac{s}{n+g+\delta} = \frac{0.2}{0.01+0.01+0.03} = 4$$

where the subscript "ini" tells us that this value belongs to an economy that retains its initial parameter values into the future. Thus 69 years into the future, at t=69:

$$\left(\frac{Y_69}{L_69}\right)_i ni = (140000)e^{(0.01)(69)} = (140000)(1.9937) = 279120$$

Now let us consider an alternative scenario in which output per worker is the same in year 0 but in which the efficiency of labor growth rate g is a higher rate. Suppose $g_{alt} = g_{ini} + \Delta g$, with the subscript "alt" reminding us that this parameter or variable belongs to the alternative scenario just as "ini" reminds us of the initial scenario or set of values. How do we forecast the growth of the economy in an alternative scenario—in this case, in an alternative scenario in which $\Delta g = 0.02$.

The first thing to do is to calculate the balanced growth path steady-state capital-output ratio in this alternative scenario. Thus we calculate:

$$\left(\frac{K}{Y}\right)^{*}_{alt} = \frac{s}{n + g_{ini} + \Delta g + \delta} = \frac{0.20}{0.01 + 0.01 + 0.02 + 0.03} = \frac{0.20}{0.07} = 2.857$$

The steady-state balanced growth path capital-output ratio is much lower in the alternative scenario than it was in the initial scenario: 2.857 rather than 4. The capital-output ratio, of course, does not drop instantly to its new steady-state value. It takes time for the transition to occur.

While the transition is occurring, the efficiency of labor in the alternative scenario is growing at not 1% but 3% per year. We can thus calculate the alternative scenario balanced growth path value of output per worker as:

$$\left(\frac{Y_t}{L_t}\right)^{*}_{alt} = \left(\frac{K}{Y}\right)^{*\theta}_{alt} E_0 e^{(0.01+0.02)t}$$

And in the 69th year this will become:

$$\left(\frac{Y_{69}}{L_{69}}\right)^{*}_{alt} = (2.857)(35000)e^{(0.03)(69)} = 792443$$

How good would this balanced growth-path value be as an estimate of the actual behavior of the economy? We know that a Solow growth model economy closes a fraction $(1-\alpha)(n+g+\delta)$ of the gap between its current position and its steady-state balanced growth path capital-output ratio each period. For our parameter values $(1-\alpha)(n+g+\delta)=0.035$. That gives us about 20 years as the period needed to converge halfway to the balanced growth path. 69

years is thus about 3.5 such halvings of the gap—meaning that the economy will close 9/10 of the way. Thus assuming the economy is on its alternative scenario balanced growth path in year 69 is not a bad assumption.

But if we want to calculate the estimate exactly? 820752.

The takeaways are three:

For these parameter values, 69 years are definitely long enough for you to make the assumption that the economy has converged to its Solow model balanced growth path. One year no. Ten years no. Sixty-nine years, yes.

Shifts in the growth rate g of the efficiency of labor do, over time, deliver enormous differentials in output per worker across scenarios.

The higher efficiency of labor economy is, in a sense, a less capital intensive economy: only 2.959 years' worth of current production is committed to and tied up in the economy's capital stock in the alternative scenario, while 4 years' worth was tied up in the initial scenario. But the reduction in output per worker generated by a lower capital-output ratio is absolutely swamped by the faster growth of the efficiency of labor, and thus the much greater value of the efficiency of labor in the alternative scenario comes the 69th year.

2.4.3.5. Shifts in the Saving Rate s

The Most Common Policy and Environment Shock: Shifts in labor force growth rates do happen: changes in immigration policy, the coming of cheap and easy contraception (or, earlier, widespread female literacy), or increased prosperity and expected prosperity that trigger "baby booms" can all have powerful and persistent effects on labor force growth down the pike. Shifts in the growth of labor efficiency growth happen as well: economic policy disasters and triumphs, countless forecasted "new economies" and "secular stagnations", and the huge economic shocks that were the first and second Industrial Revolutions—the latter inaugurating that global era of previously unimagined increasing prosperity we call modern economic growth—push an economy's labor efficiency growth rate g up or down and keep it there.

Nevertheless, the most frequent sources of shifts in the parameters of the Solow growth model are shifts in the economy's saving-investment rate. The rise of politicians eager to promise goodies—whether new spending programs or tax cuts — to voters induces large government budget deficits, which can be a persistent drag on an economy's saving rate and its rate of capital accumulation. Foreigners become alternately overoptimistic and overpessimistic about the value of investing in our country, and so either foreign saving adds to or foreign capital flight reduces our own saving-investment rate. Changes in households' fears of future economic disaster, in households' access to credit, or in any of numerous other factors change the share of household income that is saved and invested. Changes in government tax policy may push after-tax returns up enough to call forth additional savings, or down enough to make savings seem next to pointless. Plus rational or irrational changes in optimism or pessimism—what John Maynard Keynes labelled the "animal spirits" of individual entrepreneurs, individual financiers, or bureaucratic committees in firms or banks or funds all can and do push an economy's savings-investment rate up and down.

Analyzing a Shift in the Saving Rate s: What effects do changes in saving rates have on the balanced-growth path levels of Y/L?

The higher the share of national product devoted to saving and gross investment—the higher is s—the higher will be the economy's balanced-growth capital-output ratio s/(n + g + δ). Why? Because more investment increases the amount of new capital that can be devoted to building up the average ratio of cap ital to output. Double the share of national product spent on gross investment, and you will find that you have doubled the economy's capital intensity, or its average ratio of capital to output.

As before, the equilibrium will be that point at which the economy's savings effort and its investment requirements are in balance so that the capital stock and output grow at the same rate, and so the capital-output ratio is constant. The savings effort of society is simply sY, the amount of total output devoted to saving and investment. The investment requirements are the amount of new capital needed to replace depreciated and worn-out machines and buildings, plus the amount needed to equip new workers who increase the labor force, plus the amount needed to keep the stock of tools and machines at the disposal of more efficient workers increasing at the same rate as the efficiency of their labor.

$$sY = (n + g + \delta)K$$

And so an increase in the savings rate s will, holding output Y constant, call forth a proportional increase in the capital stock at which savings effort and investment requirements are in balance: increase the saving-investment rate, and you double the balanced-growth path capital-output ratio:

$$\frac{K^*}{Y}_{ini} = \frac{s_{ini}}{n + g + \delta}$$

$$\frac{K^*}{Y}_{alt} = \frac{s_{ini} + \Delta s}{n + g + \delta}$$

$$\frac{K^*}{Y}_{alt} - \frac{K^*}{Y}_{ini} = \frac{\Delta s}{n + g + \delta}$$

with, once again, balanced growth path output per worker amplified or damped by the dependence of output per worker on the capital-output ratio:

$$\frac{Y^*}{L} = \frac{K^*}{Y} E$$

Analyzing a Shift in the Saving-Investment Rate: An Example: To see how an increase in the economy's saving rate s changes the balanced-growth path for output per worker, consider an economy in which the parameter θ=2 (and α=⅔, the rate of labor-force growth n is 1 percent per year, the rate of labor efficiency growth g is 1.5 percent per year, and the depreciation rate δ is 3.5 percent per year.

Suppose that the saving rate s, which had been 18 percent, suddenly and permanently jumped to 24 percent of output.

Before the increase in the saving rate, when s was 18 percent, the balanced-growth equilibrium capital-output ratio was:

$$\frac{K^*}{Y}_{ini} = \frac{s_{ini}}{n + g + \delta} = \frac{0.18}{0.06} = 3$$

After the increase in the saving rate, the new balanced-growth equilibrium capital- output ratio will be:

$$\frac{K^*}{Y}_{alt} = \frac{s_{ini} + \Delta s}{n + g + \delta} = \frac{0.24}{0.06} = 4$$

We see, with a value of θ=2, that balanced-growth path output per worker after the jump in the saving rate is higher by a factor of $(4/3)^2 = 16/9$, or fully 78 percent higher.

Just after the increase in saving has taken place, the economy is still on its old, balanced-growth path. But as decades and generations pass the economy converges to its new balanced-growth path, where output per worker is not 9 but 16 times the efficiency of labor. The jump in capital intensity makes an enormous differ ence for the economy's relative prosperity.

Note that this example has been constructed to make the effects of capital intensity on relative prosperity large: The high value for θ means that differences in capital intensity have large and powerful effects on output-per-worker levels.

But even here, the shift in saving and investment does not permanently raise the economy's growth rate. After the economy has settled onto its new balanced-growth path, the growth rate of output per worker returns to the same 1.5 percent per year that is g, the growth rate of the efficiency of labor.

2.5. The Solow-Malthus Model

Two major changes to the Solow model are needed in order to make it useful for making sense of the pre-industrial past. The first is to make labor efficiency depend on the scarcity of resources. The second is to make the rate of population and labor force growth depend on the economy's prosperity. We call the changed model that results from these changes the "Solow-Malthus" model.

2.5.1. Basics

2.5.1.1. Population, Resource Scarcity, and the Efficiency of Labor

Thus we first need to make efficiency of labor a function of available natural resources per worker. We do this by setting the rate of efficiency of labor growth g equal to the difference between the rate h at which economically useful ideas are generated, and the rate of population and labor force growth n divided by an effect-of-resource scarcity parameter γ, because a higher population makes natural resources per capita increasingly scarce. Therefore:

$$\frac{dE/dt}{E} = \frac{d\ \ln(E)}{dt} = g = h - \frac{n}{\gamma}$$

Thus:

$$\frac{d}{dt}\left(\frac{Y}{L}\right)^{*} = 0 \text{ ; whenever } h - \frac{n}{\gamma} = 0$$

$$n^{*mal} = \gamma h$$

is the population growth rate at which:

$$\frac{d}{dt}\left(\frac{Y}{L}\right)^{*} = 0$$

When population is growing at the rate n^{*mal}, the efficiency of labor—and thus the steady-state growth-path level of production per worker Y/L—is constant. This captures the idea that even though human technology was advancing over the ten millennia before the Industrial Revolution, living standards were not because the potential benefits from technology and organization for productivity were offset by the productivity-diminishing effects of smaller farm sizes and more costly other natural resources to feed and provide for the growing population.

2.5.1.2. Determinants of Population and Labor Force Growth

We also need to make the rate of growth of the population and labor force depend on the level of prosperity $y{=}Y/L$; on the "subsistence" standard of living for necessities y^{sub}; and also on the fraction $1/\phi$ of production that is devoted to necessities, not conveniences and luxuries, and thus enters into reproductive and survival fitness. The higher the resources devoted to fueling reproductive and survival fitness, the faster will be the rate of population growth:

$$\frac{dL/dt}{L} = \frac{d\ \ln(L)}{dt} = n = \beta\left(\frac{y}{\phi y^{sub}} - 1\right)$$

Then for population to be growing at its Malthusian rate:

$$\gamma h = \beta \left(\frac{1}{\phi}\right) \left(\frac{y}{y^{sub}} - \phi\right)$$

$$y^{*mal} = \phi y^{sub} \left(1 + \frac{n^{*mal}}{\beta}\right) = \phi y^{sub} \left(1 + \frac{\gamma h}{\beta}\right)$$

Note that these only hold for poor populations—one that have not gone through the demographic transition. When populations grow rich and literate enough—and when women acquire enough social power—human societies undergo the demographic transition: women limit their pregnancies to the number of children they desire, confident that they will pretty much all survive to outlive them. Beyond a certain income level, equation (5.4) no longer holds. But it did hold up until well after the start of the Industrial Revolution.

2.5.2. The Full Malthusian Equilibrium

Then with these added to our Solow growth model to turn it into the Solow-Malthus model, we can calculate the full Malthusian equilibrium for a pre-industrial economy. We can determine the log-level *ln(E)* of the efficiency of labor:

$$\ln(E) = \ln(H) - \frac{\ln(L)}{\gamma}$$

Then since:

$$y^{*mal} = \left(\frac{s}{\gamma h + \delta}\right)^{\theta} E$$

$$\ln(\phi) + \ln\left(y^{sub}\right) + \ln\left(1 + \frac{\gamma h}{\beta}\right) = \theta \ln(s) - \theta \ln(\gamma h + \delta) + \ln(E)$$

The population and labor force in the full Malthusian equilibrium will be:

$$\ln(L_t^{*mal}) = \gamma\left[\ln(H_t) - \ln(y^{sub})\right] + \gamma\theta\left(\ln(s) - \ln(\delta)\right) - \gamma\ln(\phi) + \left(-\gamma\theta\ln(1 + \gamma h/\delta) - \gamma ln\left(1 + \frac{\gamma h}{\beta}\right)\right)$$

2.5.3. Understanding the Malthusian Equilibrium

Thus to analyze the pre-industrial Malthusian economy, at least in its equilibrium configuration:

- Start with the rate h at which new economically-useful ideas are being generated and with the responsiveness β of population growth to increased prosperity.
- From those derive the Malthusian rate of population growth: $n^{*mal} = \gamma h$
- Then the Malthusian standard of living is: $y*mal = \phi y^{sub}(1+\gamma h/\beta)$
- And the Malthusian population is:

$$L_t^{*mal} = \left[\left(\frac{H_t}{y^{sub}}\right)\left(\frac{s}{\delta}\right)^{\theta}\left(\frac{1}{\phi}\right)\left[\frac{1}{(1+\gamma h/\delta)^{\theta}}\frac{1}{(1+\gamma h/\beta)}\right]\right]^{\gamma}$$

Thus at any date t, the Malthusian-equilibrium population is:

1. the current level H_t of the valuable ideas stock divided by the (sociologically determined, by, for example western European delayed female marriage patterns, or lineage-family control of

reproduction by clan heads) Malthusian-subsistence income level y^{sub} consistent with a stable population on average, times

2. the ratio between the savings-investment rate s and the depreciation rate δ, raised to the parameter θ which governs how much an increase in the capital-output ratio raises income—with a higher θ, factors like the rule of law, imperial peace, and a culture of thrift and investment that potentially boost the economy's capital stock will matter more, and can generate "efflorescences"—times
3. one over the conveniences-and-luxuries parameter ϕ—it drives a wedge between prosperity and subsistence as spending is diverted categories that do not affect reproduction, such as middle-class luxuries, upper-class luxuries, but also the "luxury" of having an upper class, and the additional conveniences of living in cities and having trade networks that can spread plagues—times
4. two nuisance terms near one, which depend on how much the level of population must fall below the true subsistence level at which population growth averages zero to generate the (small) average population growth rate that produces growing resource scarcity that offsets the (small) rate of growth of useful ideas. All this
5. raised to the power γ that describes how much more important ideas are than resources in generating human income and production.

(1) is the level of the stock of *useful ideas* relative to the requirements for subsistence. (2) depends on how the rule of law and the rewards to thrift and entrepreneurship drive savings and investment, and thus the division of labor. (3) depends on how society diverts itself from nutrition and related activities that aim at boosting reproductive fitness and, instead, devotes itself to conveniences and luxuries—including the "luxury" of having an upper class, and all the conveniences of urban life. (4) are constant, and are small. And (5) governs how productive potential is translated into resource scarcity-generating population under Malthusian conditions.

And recall the full Malthusian equilibrium standard of living:

$$y^{*mal} = \phi y^{sub} \left(1 + \frac{\gamma h}{\beta}\right)$$

This level of income is:

1. The luxuries-and-conveniences parameter ϕ, times
2. The level of subsistence y^{sub}, times
3. The (small and constant) nuisance parameter $1+\gamma h/\beta$ needed to generate average population growth $n^{*mal} = \gamma h$ sufficient for increasing resource scarcity to offset technological progress and so hold productivity and incomes at their Malthusian-equilibrium constant levels.

2.5.4. Implications for Understanding Pre-Industrial Civilizations

Production per worker and thus prosperity are thus primarily determined by (a) true subsistence, (b) the wedge between prosperity and reproductive fitness produced by spending on conveniences and luxuries that do not impact reproductive success, plus a minor contribution by (c) the wedge above subsistence needed to generate population growth consonant with the advance of knowledge and population pressure's generation of resource scarcity.

With this model, we can investigate broader questions about the Malthusian Economy—or at least about the Malthusian model, with respect to its equilibrium:

- How much does the system compromise productivity, both static and dynamic, to generate inequality?
- How would one rise in this world—or avoid losing status relative to your ancestors?
- How does the system react to shocks?:
    - like a sudden major plague—like the Antonine plague of 165, the St. Cyprian plague of 249, or the Justinian plague of 542—that suddenly and discontinuously pushes population down sharply…
    - like the rise of a civilization that carries with it norms of property and law and commerce, and thus a rise in the savings-investment rate s…
    - like the rise of an empire that both creates an imperial peace, and thus a rise in the savings-investment rate s, and that also creates a rise in the taste for luxuries ϕ (and possibly reduces biological subsistence y^{sub} as well…
    - like the fall of an empire that destroys imperial peace, and thus a fall in the savings-investment rate s, and in the taste for luxuries ϕ and possibly raises biological subsistence y^{sub} as looting, pillaging, and murdering barbarians stalk the land…
    - a shift in the rate of ideas growth…
    - a shift in sociology that alters subsistence...

The fall of an empire, for example, would see a sharp decline in the savings-investment share s, as the imperial peace collapsed, a fall in the "luxuries" parameter ϕ, as the taste for urbanization and the ability to maintain gross inequality declined, and possibly a rise in y^{sub}, if barbarian invasions, wars, and social-order breakdown significantly raised mortality from violent death.

This model provides an adequate framework—or I at least, think it is an adequate framework—for thinking about the post-Neolithic Revolution pre-Industrial Revolution economy.

2.5.5. Dynamics

2.6. Determinants of the Rate of Technological Progress

2.6.1. The Depressing Bottom Line

The rate of economic growth—first in population, and more recently in average living standards and productivity growth rates—hinges on the proportional rate of increase h in the human stock of useful ideas H. And this rate has been extraordinarily variable in the long sweep of human history.

Longest-Run Global Economic Growth (2019)

Date	ideas Level H	Total Real World Income Y (billions)	Average Real Income per Capita y (per year)	Total Human Population L (millions)		Rate of Population and Labor Force Growth n	Rate of Efficiency-of-Labor Growth g	Rate of Ideas-Stock Growth h
-68000	1.0	$0	$1,200	0.1				
-8000	5.0	$3	$1,200	2.5		0.005%	0.000%	0.003%
-6000	6.3	$6	$900	7		0.051%	-0.014%	0.011%
-3000	9.2	$14	$900	15		0.025%	0.000%	0.013%
-1000	16.8	$45	$900	50		0.060%	0.000%	0.030%
0	30.9	$153	$900	170		0.122%	0.000%	0.061%
800	41.1	$270	$900	300		0.071%	0.000%	0.035%
1500	53.0	$450	$900	500		0.073%	0.000%	0.036%
1770	79.4	$825	$1,100	750		0.150%	0.074%	0.149%
1870	123.5	$1,690	$1,300	1300		0.550%	0.167%	0.442%
2020	2720.5	$90,000	$11,842	7600		1.177%	1.473%	2.061%
2100	13474.9	$485,096	$53,900	9000	?	0.211%	1.894%	2.000%
2200	99566.8	$3,584,405	$398,267	9000	?	0.000%	2.000%	2.000%
2500	40168118.9	$1,446,052,279	$160,672,475	9000	?	0.000%	2.000%	2.000%

If the trends of the past century and a half were to continue for the next three, we would look forward to truly cray-cray levels of abundant human wealth:

Moreover, right now the divergences across national economies are as great as they have ever been, and orders of magnitude greater than they were three centuries ago or even one century ago. What insights can economists offer into these phenomena?

Unfortunately, the bottom line is that economists have little that is terribly useful to say about the proportional rate h at which the human stock H of useful and valuable ideas about technology and organization increases. It would not be too much a parody to say that economists know only four things:

1. People learn by doing: trying to produce, and succesfully producing, brings with it knowledge about how to produce more efficiently and effectively.
2. People learn by investing: a great deal of knowledge is embodied in the particular capital goods themselves produced and deployed; if you do not invest, a great deal of your knowledge remains theoretical.
3. People learn by researching and developing: focused attention on the process of developing technology can be very effective—much more so than simply relying on the side-effects of those whose major focus is on production itself
4. Knowledge is non-rival: once it is generated, it can and should be spread as widely as possible, for there is no downside for society as a whole from sharing.

But that—especially that without sound and solid quantitative estimates of the size and importance of these effects and channels—is rather thin gruel given that the growth and diffusion of useful knowledge about production and organization is the big enchilada in the process of economic growth.

Here we will focus on (3) and (4), leaving (1) and (2) for later "applications" sections of this course:

2.6.2. Knowledge Is Non-Rival

2.6.2.1. Logical Implications

Useful ideas about technology and organization are non-rival: one person's work in adding to H can rapidly benefit all—if it is allowed to spread. And attempts to keep it from spreading—to limit knowledge's distribution by somehow charging those using it a price—must violate the optimality condition that the costs imposed on people for making use of commodities reflect and match the burden that their withdrawal of the commodities from the common stock imposes on the rest of the community, for with non-rival commodities there is no such withdrawal. The insights that knowledge is key and that knowledge is non-rival are now nearly two centuries old. We can find them in in 1843:

> **Friedrich Engels** (1843): *Outlines of a Critique of Political Economy* https://www.marxists.org/archive/marx/works/1844/df-jahrbucher/outlines.htm: ‘According to the economists, the production costs of a commodity consist of three elements: the rent for the piece of land required to produce the raw material; the capital with its profit, and the wages for the labour required for production and manufacture.... [Since] capital is “stored-up labour”... two sides–the natural, objective side, land; and the human, subjective side, labour, which includes capital and, besides capital, a third factor which the economist does not think about–I mean the mental element of invention, of thought, alongside the physical element of sheer labour. What has the economist to do with inventiveness? Have not all inventions fallen into his lap without any effort on his part? Has one of them cost him anything? Why then should he bother about them in the calculation of production costs? Land, capital and labour are

> for him the conditions of wealth, and he requires nothing else. Science is no concern of his.
>
> What does it matter to him that he has received its gifts through Berthollet, Davy, Liebig, Watt, Cartwright, etc.–gifts which have benefited him and his production immeasurably? He does not know how to calculate such things; the advances of science go beyond his figures. But in a rational order which has gone beyond the division of interests as it is found with the economist, the mental element certainly belongs among the elements of production and will find its place, too, in economics among the costs of production.
>
> And here it is certainly gratifying to know that the promotion of science also brings its material reward; to know that a single achievement of science like James Watt's steam-engine has brought in more for the world in the first fifty years of its existence than the world has spent on the promotion of science since the beginning of time…

And yet indeed it was the case that mainstream economists, for generations, paid remarkably little of their attention to "inventiveness". Engels was right—at least about mainstream economists' strange neglect. (I take no stance on whether Engels was right in his belief that mainstream economists cannot see the world as it is but only illusions that are caused by our particular institutional framework and convenient to those whom our current institutional framework serves most fulsomely.) Engels was right so much so that Paul Romer received the Nobel Prize in 2018 for his attempts to bring "inventiveness" back to the center. When **Robert Solow** (1987): *Growth Theory and After* https://www.nobelprize.org/prizes/economic-sciences/1987/solow/lecture/ gave his Nobel lecture on the occasion of the earlier "economic growth" Nobel Prize, awarded in 1987, he noted that his theory had little to say about "technical change in the broadest sense" and that this was a huge flaw:

> The "neoclassical model of economic growth" started a small industry... stimulated hundreds of theoretical and empirical articles... very quickly found its way into textbooks... is what allows me to think

> that I am a respectable person to be giving this lecture today.... Gross output per hour of work in the U. S. economy doubled between 1909 and 1949; and some seven-eighths of that increase could be attributed to “technical change in the broadest sense” and only the remaining eight could be attributed to conventional increase in capital intensity.... I had expected to find a larger role for straightforward capital formation...

Solow goes on to write that his attempts to say something meaningful and important about the determinants of h largely failed:

> "embodiment", the fact that much technological progress, maybe most of it, could find its way into actual production only with the use of new and different capital equipment... [and so] a policy to increase investment would thus lead... also to a faster transfer of new technology into actual production, which would [matter much].... That idea seemed to correspond to common sense, and it still does.... If common sense was right, the embodiment model should have fit the facts significantly better than the earlier one. But it did not...

There is a literature, springing from Paul Romer's work in the 1980s, focusing on the implications of non-rivalry in the use of ideas: that one person's work in adding to $\boldsymbol{H}$ can rapidly benefit all. The first conclusion is that production must in some sense be subject to increasing returns. We know that if all material "inputs" were to double then, since the new inputs could just do the same things as the old ones, production would at least double. In knowledge production, however, pursuing the same lines of inquiry and thus making all the same discoveries twice would be silly. Doubled material inputs with doubled effort devoted to knowledge creation should therefore more than double output. An economy in which non-rival knowledge is important will therefore exhibit *scale effects*: size matters, and in a good way for growth.

The useful literature can be seen as building on this first conclusion.

2.6.2.2. Fitting the Entire Span of Human History: Michael Kremer on Growth since One Million B.C.:

Theory (1993): Non-rivalry in the use of and non-crowding in the production of useful ideas about technology and organization are the fundamental underlying assumptions of the first milestone to visit on our path through the literature:

> Michael Kremer: **Michael Kremer** (1993) *Population Growth and Technological Change: One Million B.C. to 1990* https://delong.typepad.com/files/kremer-million.pdf: The long-run history of population growth and technological change is consistent with the population implications of models of endogenous technological change... a highly stylized model in which... the growth rate of technology is proportional to total population... the Malthusian assumption that population is limited by the available technology, so that the growth rate of population is proportional to the growth rate of technology. Combining these assumptions implies that the growth rate of population is proportional to the level of population.... The prediction that the population growth rate will be proportional to the level of population is broadly consistent with the data... until recently.... If population grows at finite speed when income is above its steady state... per capita income will rise over time. If population growth declines in income at high levels of income, as is consistent with a variety of theoretical models and with the empirical evidence, this gradual increase in income will eventually lead to a decline in population growth.... As the model predicts, the growth rate of population has been propor- tional to its level over most of history.... Among technologically separate societies, those with higher initial population had faster growth rates of technology and population…

In short, two heads are better than one:

Assume that output is given by:

$$Y = Ap^{\alpha}R^{1-\alpha}$$

where A is the level of technology, p is population, and R=1 is land, normalized to one unit. Per-worker income y = Y/L therefore equals:

$$Ap^{\alpha-1}$$

Population increases above the Malthusian steady-state equilibrium level of per capita income y^* and decreases below it. Diminishing returns to labor imply that for each value of A a unique level of population, p^***,** generates income y^*:

$$p^* = \left(\frac{A}{y^*}\right)^{(1/(1-\alpha))}$$

In a larger population there will be proportionally more people lucky or smart enough to come up with new ideas. If research productivity per person is independent of population and if A affects research output the same way it affects output of goods (linearly, by definition), then the rate of change of technology will be:

$$\frac{dA}{dt} = \pi Ap$$

Take the log derivative of the population determination equation:

$$\frac{dln(p)}{dt} = \left(\frac{1}{1-\alpha}\right)\frac{dln(A)}{dt}$$

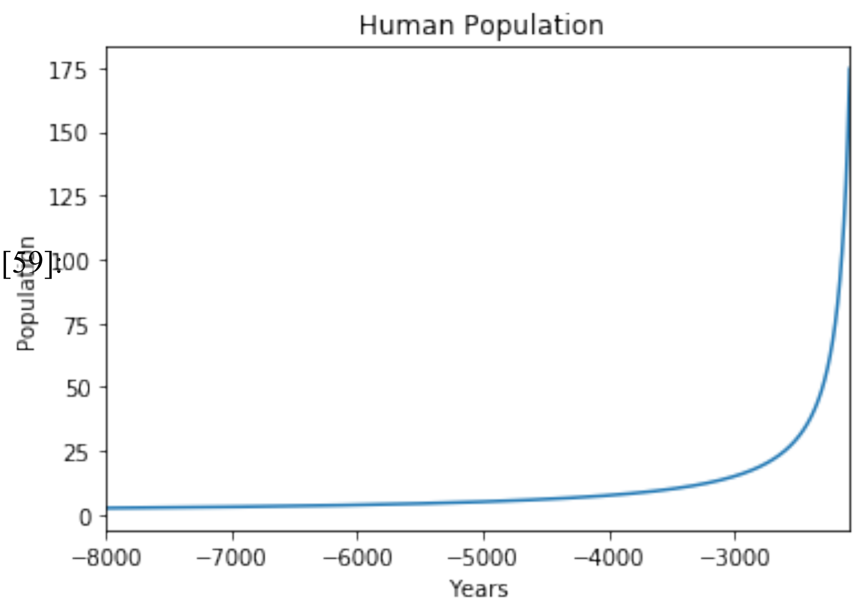
and substitute in the expression for the growth rate of technology:

$$\frac{dp}{dt} = \frac{\pi p^2}{1-\alpha}$$

to get superexponential growth of population (and total income)—as long as the Malthusian régime lasts, there is no demographic transition, .

To get an idea of what this means, let us run a computational experiment. There were 2.5 million people 10,000 years ago, at the invention of agriculture. There were 15 million people 5,000 years ago, at the invention of writing. There were 170 million people in the year 1. Let's calibrate this model to 2.5 million people in the year -8000 and 15 million people in the year -3000: a value of $\pi/(1-\alpha)=0.00006666$ serves. But that value predicts that human population would cross 170 million heading upwards not in the year 1 but in the year -2080: early in the Bronze Age.

[59]:

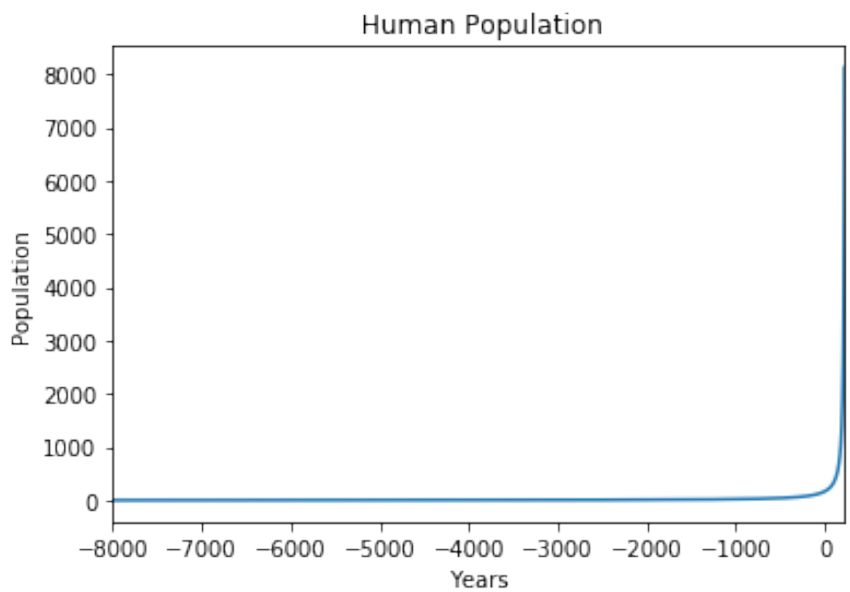


If we want to fit our three pre-1 benchmarks, we cannot have two heads being fully as good as one. So, instead, let us assume not that a 1% increase in the STEM workforce raises the rate of technological progress by 1% but rather by λ% for some parameter λ. So the dynamics for population then become:

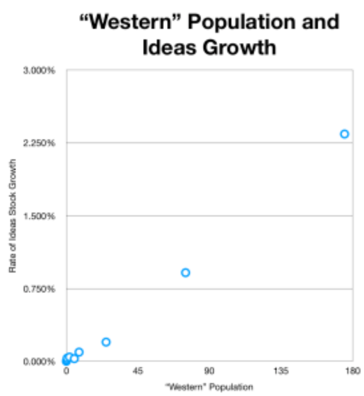
$$\frac{dp}{dt} = \frac{\pi p^{1+\lambda}}{1-\alpha}$$

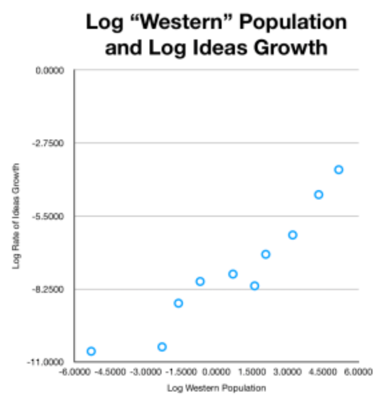
α=0.5, π=0.00003264, λ=0.8529 fit the pre-1 benchmarks well. But those benchmarks predict that the human population would have exploded in the following two centuries, and crossed rthe world's current population of 7.6 billion in the year 221.

Even if two heads are not quite as good as one—are only 1.85 times as good as one—there need to be other sources of drag in order to have kept the world from an Industrial Revolution-class breakthrough late in the Later Han, and under the late Antonine and Severan dynasties:



How Well Does This Fit Human History?: Still, all in all, Michael Kremer says: it does not fit badly badly. And, indeed, up to 1900 the rate of change of the human population is indeed roughly proportional to the square of the population—as long as we start not with the invention of agriculture but with the invention of writing, and with a hiccup as the Roman and Han empires collapsed in the second third of the first millennium. After 1900 things fall apart: increasing populations and the increasing ability of people to use technology and wealth to help their investigations do not pay dividends, either in further accelerating population growth or





in increasing the rate of growth of global incomes. Nevertheless, two heads are somewhat, if not linearly better than one.

Or maybe not: surely the effective STEM labor force depends on means of knowledge recording and communication. And it is not foolish to expect *ex ante* that there would be some diminishing returns from exhaustion of low-hanging fruit at some point. We do seem to see a jump up in growth with the invention of writing, and cities. Shouldn't we also see a jump up with the alphabet? Shouldn't we also see a jump up with the invention of printing?

Perhaps the effects of the picking of the low-hanging fruit in exhausting opportunities and slowing growth civilization-wide are visible in the slowdown after the year one. Perhaps the effective STEM labor force gets big bumps up with the alphabet and with printing that together, in the large, offset this exhaustion. Clearly, however, two heads are better than one will not suffice to understand the relative constancy of global economic growth rates since the coming of modern economic growth around 1870, or even the failure of Roman and Han civilization to usher in an industrial revolution.

2.6.2.3. Chad Jones on R & D-Based Models of Economic Growth

Can we preserve the insights that ideas are non-rival and that technology is the ballgame and still understand why growth did not accelerate faster and bring us an Industrial Revolution early in the first millennium, and, in fact, has not further accelerated since the late 1800s? Chad Jones believes we can, and he lays out his case in **Charles I. Jones** (1995): *R&D-Based Models of Economic Growth* https://delong.typepad.com/files/jones-r--d.pdf:

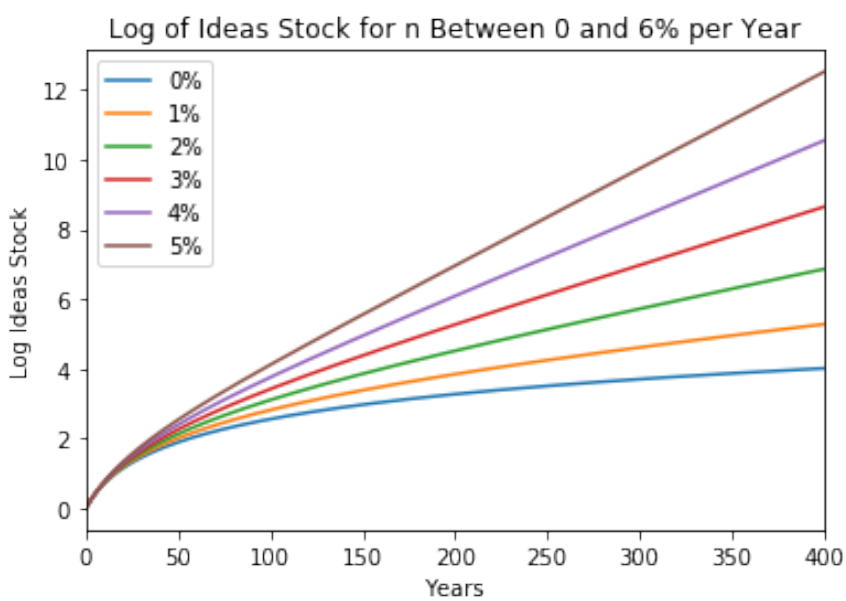
> The prediction of permanent scale effects on growth from the R&D equation means that the models of Romer/Grossman-Helpman/Aghion-Howitt and others are all easily rejected.... However, the R&D-based models [remain] intuitively very appealing.... [Is there] a way to maintain the basic structure of these models while eliminating the prediction of [permanent] scale effects [on the rate of growth?]…

Jones's answer is "yes". Jones accomplishes this by building a basic model that has both (a) an "as the low-hanging technological fruit is picked, maintaining the same proportional growth rate for the ideas stock H becomes harder" effect (the parameter $\phi<1$); and (b) an "as the STEM workforce increases, researchers tend to step on each others' toes and get in each others' way" effect (the parameter λ<1):

$$\frac{dH}{dt} = \pi L_{stem}^{\lambda} H^{\phi}$$

$$\frac{dH/dt}{H} = \pi L_{stem}^{\lambda} H^{\phi - 1}$$

To gain some intuition, let's consider six different economies in which the rate of growth n of the STEM labor force varies from 0 to 6% per year, in which the initial levels of both the ideas stock H_0 and the STEM labor force L_{stem0} are set at 1, and let us set the R&D crowding parameter λ=0.5, and, just to get striking results, the exhaustion of low hanging fruit parameter at the very low level of ϕ=0.1. Look out at the evolution of the log of the ideas stock for 400 years:



What is going on here? We can see from the constancy of the slopes on the right hand side of this log graph that the ideas stock $\boldsymbol{H}$ is heading for some steady-state growth rate. That steady-state is higher the higher is the rate of growth of the STEM labor force. And for that convergence to a constant growth rate to happen, in the long run the increase in the effective STEM labor force

$$L^{\lambda}_{STEM}$$

has to be exactly offset by diminishing returns to innovative effort:

$$\delta H^{\phi-1}$$

Thus along the ideas-stock steady-state balanced-growth path it must be true that:

$$\lambda \frac{1}{L_{stem}} \frac{dL_{stem}}{dt} = (1-\phi)\frac{dH/dt}{H}$$

$$\lambda n_{stem} = (1-\phi)h^*$$

The level of ideas $\boldsymbol{H^*}$ at which that growth rate h^* would be attained is characterized by:

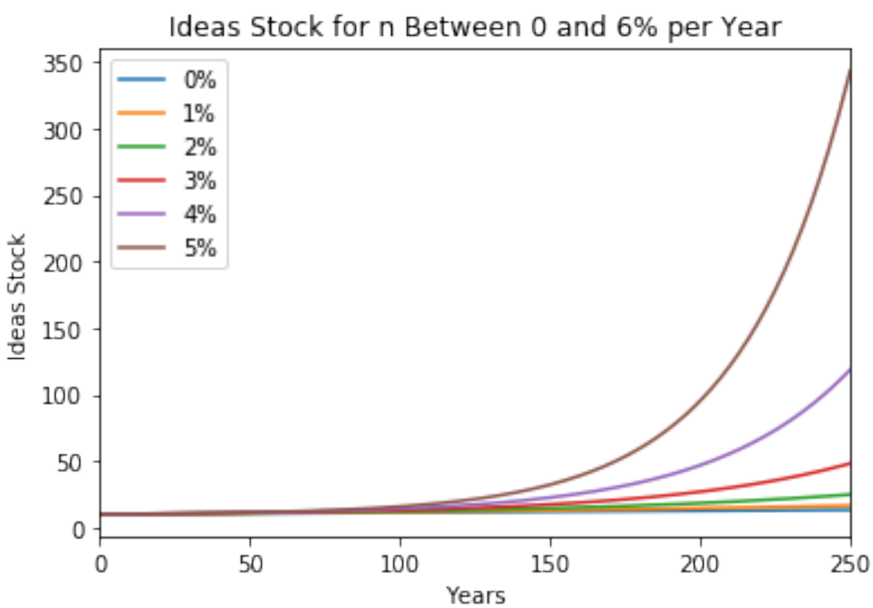
So in Jones's model $\boldsymbol{H}$ grows more rapidly than its asymptotic exponential growth rate h^* until it closes in on the value of $\boldsymbol{H^*}$ that characterizes the Jones steady-state balanced-growth path:

$$H^* = \left(\frac{\pi(1-\phi)}{\lambda}\right)^{1/(1-\phi)} \left(\frac{1}{n_{stem}}\right)^{1/(1-\phi)} L_{stem}^{\lambda/(1-\phi)}$$

And then the growth rate in the steady-state balanced-growth path knowledge stock is characterized by:

$$h^* = \left(\frac{\lambda}{1 - \phi} \right) n_{stem}$$

Thus the rate of growth of the ideas stock along the steady-state balanced-growth growth path will be proportional to the rate of growth of the STEM labor force, with constant of proportionality $\lambda/(1-\phi)$ (the degree to which more researchers step on one anothers' toes, divided by how important it is that the low-hanging innovation fruit has already been picked). The level of the ideas stock along the steady-state balanced-growth path will vary inversely with the rate of growth n of the STEM labor force raised to the power $1/(1-\phi)$, and directly with the level L_{stem} of the STEM labor force raised to the power $\lambda/(1-\phi)$.



If $H<H^*$, then H is growing faster than h^*, and the scale variable

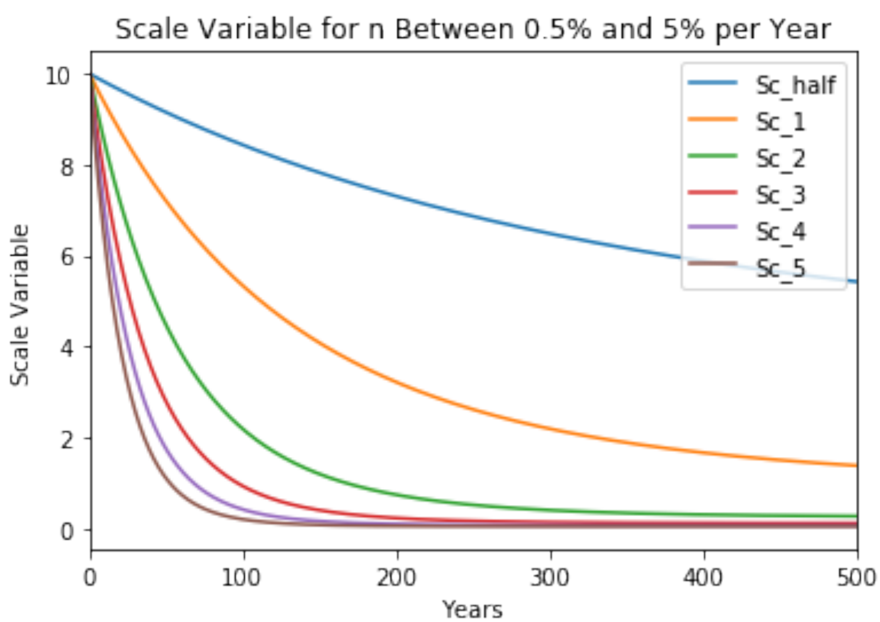
$$\frac{H}{L_{stem}^{\lambda/(1-\phi)}}$$

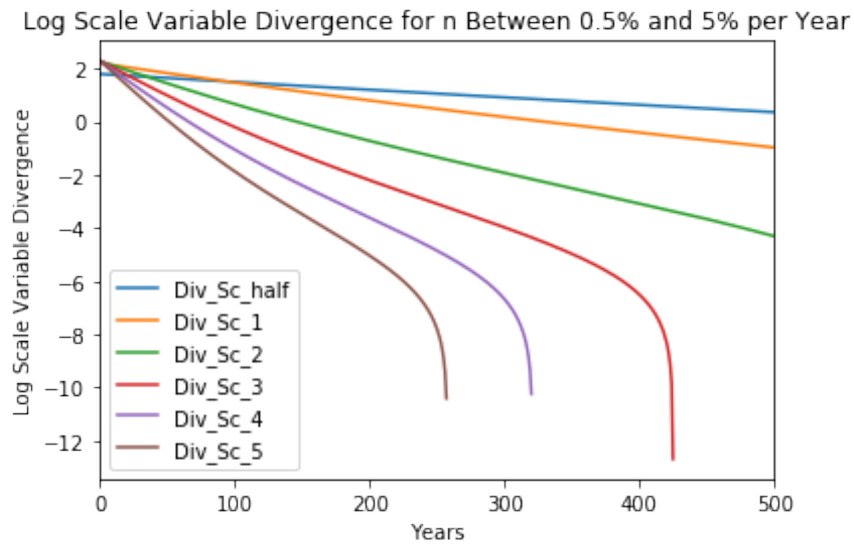
is rising.

And, indeed, looking at the levels of the ideas stock over 150 years reveals, first, initial superexponential growth; that growth rate then declines until the growth rate asymptotes (for n>0) to merely exponential growth at the rate:

$$h^* = \lambda n_{stem}/(1 - \phi)$$

How fast does this Jones model converge to its steady-state balanced-growth path with its constant rate of increase h^* in the ideas stock? To understand this, we need to look at our scale variable:





Notes and Musings: Growth of the STEM Labor Force:

25 bachelor's degrees per 1000 23 year olds in 1900...

300 bachelor's degrees per 1000 23 year olds today...

60-fold multiplication in college graduates in the U.S....

20-fold multiplication in h since 1870

$$\lambda/(1-\phi) = 1/3$$

Inflection points in the effective STEM workforce

- writing
- printing
- formal education